Exercise 1

Suppose the decision maker's preferences over monetary outcomes are described by the utility function $u(x) = \sqrt{x}$. Consider two lotteries:

- Lottery L1 pays 100 Euro with a probability of 6% and zero otherwise
- Lottery L1 pays 200 Euros with probability 3% and zero otherwise

Assuming the decision makers is an Expected Utility maximizer, which lottery does she prefer?

Exercise 2

Next years, the market for basketball shoes will be affected by an unpredictable positive or negative demand shock which is going to generate one of these two outcomes:

- Outcome O1: the market shrinks, reducing sectors' profit margins to zero
- Outcome O2: the size of the market increases or remains the same, granting a positive and significant return on investment

Consider an investor whose task is to decide whether to invest in this market of not. The agent has the choice between two actions:

- Action A1: invest
- Action A2: not invest

We assume that

- If the market shrinks (O1) the best thing to do is not to invest (A2)
- If the market increases or remains the same (O2) the best thing to do is to invest (A1)
- If the investment is performed (A1) the best outcome is outcome O2
- If the investment is not performed (A2) the best outcome is O1
- Investing in market expansion or stationary state is better than non investing in market contraction.
- Investing in market contraction is equivalent to non investing in market expansion or stationary state.

Suppose further that if the decision maker believes that the probability of outcome O1 is .8, then she is indifferent between investing and not-investing.

Build an utility function $u(A, O)$ for the decision maker, assigning a positive real number to each action conditional to each outcome, compatible with the assumptions above.

Show that if the decision maker believes that the probability of outcome O1 is .4, she will invest in the production of basketball shoes.

Exercise 3

Consider the following one-parameter family of utility functions defined over non negative amounts of money

$$u_a(x) = \frac{x}{x+a} \quad \text{with} \quad a > 0 \; .$$

Prove that $u_a(x)$ is increasing and concave. What is the most risk averse utility function between $u_{10}(x)$ and $u_{20}(x)$?

Assume that there are two investment opportunities. The first is a risk-less security, giving a gross return of 1.2 Euro in period 2 for each Euro invested in period 1. The second is a risky asset which pays, for each Euro invested in period 1, a yield of 2.4 Euro in period 2 with probability 0.8, and zero with probability 0.2. Supposing that the agent is an Expected Utility maximizer, her endowment in period 1 is equal to 3000 Euro, and the argument in her utility function is the wealth in period 2, how much does she invest in the risky security if her utility function is $u_{36}(x)$? And if her utility function is $u_{18}(x)$?

Giulio Bottazzi, Paris 28/023/2011