Training Exercises

Sequences

Ex. 1 *Compute* $\lim_{n\to\infty} a_n$ *for the following sequences*

$$a_n = \sqrt[n]{n} \qquad a_n = \frac{n}{c^n} \quad c > 1 \qquad a_n = \frac{n}{n+1} \qquad a_n = x^n \quad x \in \mathbb{R}$$

$$a_n = \sqrt[n]{2^n + 3^n} \qquad a_n = \frac{\sin n}{\sqrt{n}} \qquad a_n = \sqrt[n]{n^\alpha} \quad \alpha > 0 \qquad a_n = \sqrt{n+1} - \sqrt{n}$$

Ex. 2 *Consider the sequence defined as*

$$\begin{cases} a_1 &= 0 \\ a_{k+1} &= \sqrt{2 + a_k} \quad \textit{for} \quad k > 1 \end{cases}$$

Compute $\lim_{k\to\infty} a_k$.

Limits

Ex. 3 *Compute the following limits*

$$\lim_{x\to 0^+} \frac{2}{1 + e^{-1/x}} \qquad \lim_{x\to+\infty} \frac{2x+1}{x-1} \qquad \lim_{x\to 1} \frac{x + 1 - 2\sqrt{x}}{(x-1)^2}$$

$$\lim_{x\to+\infty} \frac{2x^2 - 5 + \sqrt{x^4 - 3x - 1}}{x - 1 + \sqrt{x^4 + x - 2}} \qquad \lim_{x\to+\infty} \sqrt{x^2 + 9x + 3} - x$$

$$\lim_{x\to 3^+} \frac{x-3}{\sqrt{x} - \sqrt{3}} \qquad \lim_{x\to+\infty} \frac{\log(1 + 2e^x)}{\sqrt{1 + x^2}} \qquad \lim_{x\to+\infty} \frac{x+4}{x^2 + x + 5}$$

$$\lim_{x\to+\infty} \sqrt{x^2 + 4} - x \qquad \lim_{x\to 0} x^2 \cos\frac{1}{x} \qquad \lim_{x\to 0} x^2 \sin\frac{1}{x} \qquad \lim_{x\to 3} \frac{x^2 - 9}{x - 3}$$

$$\lim_{x\to 2} \frac{x-2}{\sqrt{2x^2 + 1} - 3} \qquad \lim_{x\to 0} \frac{\sin(2x)\sin(3x)}{x^2}$$

Ex. 4 *Compute, if they exist, the following limits*

$$\lim_{x\to+\infty} \frac{\sqrt{9x^2+4x+5}}{x} \qquad \lim_{x\to-\infty} \frac{\sqrt{9x^2+4x+5}}{x}$$

$$\lim_{x\to0} \frac{\sqrt{5x^2+x^4}}{x} \qquad \lim_{x\to0} \frac{|\sin x|}{x}$$

$$\lim_{x\to0^+} \frac{\sqrt{x^2+x^3}}{x} \qquad \lim_{x\to0^-} \frac{\sqrt{x^2+x^3}}{x} \qquad \lim_{x\to0} \frac{\sqrt{x^2+x^3}}{x}$$

Ex. 5 *Compute the following limits, using de L'Hôpital's rule*

$$\lim_{x\to0} \frac{x-\sin x}{x^3} \qquad \lim_{x\to0} \frac{\tan x - x}{x^3}$$

$$\lim_{x\to1} \frac{x^3-3x+2}{x^3+x^2-5x+3} \qquad \lim_{x\to-2} \frac{x^2+3x+2}{2x^2-8}$$

Ex. 6 *Compute the following limits*

$$\lim_{x\to1} \frac{\sqrt{1+3x}-2}{x-1} \qquad \lim_{x\to-1} \frac{2x^2+7x+5}{3x^2+5x+2}$$

$$\lim_{x\to0} \frac{1-\cos(4x)}{x^2} \qquad \lim_{x\to2} \frac{x^2-4}{x-2} \qquad \lim_{x\to1} \frac{x^{1/3}-1}{x-1}$$

$$\lim_{x\to+\infty} \frac{\cos(3x)}{x} \qquad \lim_{x\to0} x^2\left(1+\cos\frac{1}{x}\right) \qquad \lim_{x\to+\infty} \frac{\cos x^2}{x}$$

$$\lim_{x\to0} \sqrt{x}\sin x\cos x \qquad \lim_{x\to0} \frac{\sin(-5x)}{7x} \qquad \lim_{x\to0} \frac{\cos x - 1}{\sin^2 x}$$

Functions in $\mathbb{R}$

Ex. 7 *Determine the domain of the following functions*

$$f(x) = (x^3 + 1)^{-1} \qquad f(x) = \sqrt{1 - \frac{1}{x}} \qquad f(x) = \sqrt{5 - x} + \frac{1}{\sqrt{x + 1}}$$

$$f(x) = \sin x + \tan x \qquad f(x) = \log(1 - x^2) \qquad f(x) = \sqrt{4 - x^2}$$

Ex. 8 *For the following functions study the domain, limits at the extremes of the domain, asymptots, local and global extrema, concavity. Draw a graph of the function.*

$$f(x) = 4x - x^2 + 5 \tag{1}$$

$$f(x) = (2 + \cos^2 x) \sin x \tag{2}$$

$$f(x) = xe^{-x} \tag{3}$$

$$f(x) = x + \frac{1}{x} \tag{4}$$

$$f(x) = x - \log x \tag{5}$$

$$f(x) = 3x^2 - 4x - 5 \tag{6}$$

$$f(x) = x - 1 + \sin x \tag{7}$$

$$f(x) = \frac{1}{x^2 + 4} - 3x + 4 \tag{8}$$

$$f(x) = \log\left(\frac{x^{1/3}}{3x - 1}\right) \tag{9}$$

$$f(x) = 2x + 3(e^x - 2)^{2/3} \tag{10}$$

$$f(x) = \left| x - \frac{x}{\log x} \right| \tag{11}$$

$$f(x) = 1 - (\log x)^3 \tag{12}$$

$$f(x) = e^{1/x} + \frac{\sin x}{x} \tag{13}$$

$$f(x) = xe^{\frac{x-1}{x+3}} \tag{14}$$

$$f(x) = \sqrt{x^2 + 1} - \sqrt{x} \tag{15}$$

$$\log(|x + 1| + e^x) \tag{16}$$

$$f(x) = x^5 - 10x^3 + 25x \tag{17}$$

$$f(x) = \frac{x + 1}{x^2 - 9} \tag{18}$$

$$f(x) = \frac{x^2 + 1}{x^2 - 1} \tag{19}$$

$$f(x) = \frac{x^2 + 1}{(x - 2)(x - 4)} \tag{20}$$

$$f(x) = \frac{x}{x^2 + 1} \tag{21}$$

$$f(x) = \frac{x^2 - 3}{x - 2} \tag{22}$$

$$f(x) = \frac{x^2 + 3x - 3}{x - 1} \tag{23}$$

$$f(x) = x^2 - 7x + 6 \tag{24}$$

$$f(x) = x^3 - x \tag{25}$$

$$f(x) = \cos(2x) \tag{26}$$

$$f(x) = \log|x| \tag{27}$$

$$f(x) = \frac{x-1}{x+2} \tag{28}$$

$$f(x) = \sqrt{x^2+1} \tag{29}$$

$$f(x) = \frac{1}{x^2+1} \tag{30}$$

Ex. 9 *Determine a, b, c and d such that the function $f(x) = ax^3 + bx^2 + cx + d$ passes through the points $(0, 3)$ and $(2, 5)$ and has stationary points in $x = 1/3$ and $x = 1$.*