# Exam of "Decision under uncertainty", Pisa 27/06/2011

Solve the following two exercises, motivating your answers. Please notice that: you can keep with you the notes and the textbook of the course, but no other books are allowed; you can use a pocket/scientific calculator, but computers or other advanced devices able to perform symbolic computation are not allowed; at the end of the exam, you are required to deliver all the paper (draft included) to the teacher.

## Exercise 1

Consider the following utility function defined over non negative amounts of money

$$u(x) = x - \frac{1}{x} \, . \tag{1}$$

Prove that $u(x)$ is increasing and concave and compute its relative risk aversion coefficient.

Consider the following two lotteries:

- for any dollar invested in period 1, in period 2 lottery $L_1$ pays 3 dollars with probability 1/3 and one third (1/3) of a dollar otherwise;
- for any dollar invested in period 1, in period 2 lottery $L_2$ pays 4 dollars with probability 1/4 and one forth (1/4) of a dollar otherwise.

The decision maker uses (1) to evaluate amounts of money in period 2. Prove that given an endowment of 4, she will prefer to invest the entire amount in lottery $L_2$ rather than keeping this amount for period 2. Find the initial endowment $x$ for which she is indifferent between keeping the amount $x$ for period 2 or investing it entirely in lottery $L_1$.[1]

Consider a lottery paying 5 with probability 1/2 and 1 otherwise. Find the certainty equivalent of this lottery if preferences are described by (1).

## Exercise 2

A forthcoming scientific innovation is likely to bring significant reduction in the production costs of firm A and B, generating higher prospective revenues. The decision maker has to choose in which firm to invest. She is an expected utility maximizer with linear utility depending on future revenues. Given that

- for both firms, if the scientific innovation is successful, then the generated revenues are higher;
- if the scientific innovation does not have success, firm A will generate a higher revenues than firm B;
- if the scientific innovation is successful, revenues of firm A will double;
- if the probability of success of the scientific innovation is $\pi = 1/10$, then investing in the two firms is equivalent;

assign appropriate utility levels to the following action-outcomes pairs: investing in A if the scientific innovation is successful, investing in B if the scientific innovation is successful, investing in A if the scientific innovation is not successful and investing in B if the scientific innovation is not successful.

If the probability of a scientific success is estimated to be 1/5, how will the decision maker splits her investment among the two firms?

[1] Hint: keeping the endowment for period 2 means investing in a lottery that pays one dollar for each dollar invested with probability one.