## Exam of "Decision under uncertainty", Pisa 14/07/2011

Solve the following two exercises, motivating your answers. Notes and the textbook of the course, but no other books, are allowed; a pocket/scientific calculator is permitted, but devices able to perform symbolic computation are not; at the end of the exam you are required to deliver all the paper (draft included) to the teacher. Organize the material appropriately, indicate the exercise you are solving and explain your computations. Unclear solutions will be considered wrong.

## Exercise 1

Consider the utility function

$$u(x) = (1+x)^{1/3}$$

defined over positive amounts of money. Prove that u(x) is increasing and concave. Compute its absolute risk aversion coefficient.

Consider the following two lotteries

- lottery  $L_1$  pays 7 units of money with probability 1/6, 26 units with probability 1/6 and zero otherwise;
- lottery  $L_2$  pays 7 units of money with probability 2/3 and zero otherwise.

Which lottery is preferred by an Expected Utility maximizer whose preferences are described by u(x)?

Consider a lottery  $L_3$  whose payoff is a real number randomly extracted with uniform probability in the closed interval [0,7]. This means that any  $x \in [0,7]$  has the same probability to be extracted. Compute the certainty equivalent of  $L_3$  under u(x). Hint: The probability distribution associated with the lottery  $L_3$  is F(x) = 0 for x < 0, F(x) = x/7 for  $x \in [0,7]$  and F(x) = 1 for x > 7.

Assume that there are two investment opportunities. The first is a risk-less security, giving a gross return of 1.1 monetary units in period 2 for each monetary unit invested in period 1. The second is a risky asset which pays, for each monetary unit invested in period 1, a yield of 2.2 monetary units in period 2 with probability 0.8, and zero with probability 0.2. Consider an Expected Utility maximizer investor with an endowment of 10 monetary units in period 1 and preferences about amounts of money in period 2 described by u(x). Prove that if this investor is allowed to split her initial endowment among the two securities, she will invest a positive amount of money in both. Find the share of the initial endowment optimally invested in the risky asset.

## Exercise 2

Consider the following two lotteries:

- lottery  $L_1$  pays 10 with probability 1/2 and 5 otherwise;
- Lottery  $L_2$  pays 15 with probability 1/3 and 5 otherwise.

Assume that  $L_1 \succ L_2$ , that is the decision maker prefers lottery  $L_1$  to  $L_2$ . Then consider these two new lotteries:

- lottery  $L'_1$  pays 15 with probability 1/6, 10 with probability 1/4 and 5 otherwise;
- lottery  $L'_2$  pays 15 with probability 1/4, 10 with probability 1/8 and 5 otherwise.

Prove that if the decision maker's preferences are consistent with the Expected Utility Theory, than  $L'_1 \succ L'_2$ , that is lottery  $L'_1$  is preferred to lottery  $L'_2$ . Notice that the outcomes of different lotteries are assumed independent.