

Exam of “Decision under uncertainty”, Pisa 12/09/2011

Solve the following two exercises, motivating your answers. Notes and the textbook of the course, but no other books, are allowed; a pocket/scientific calculator is permitted, but devices able to perform symbolic computation are not; at the end of the exam you are required to deliver all the paper (draft included) to the teacher. Organize the material appropriately, indicate the exercise you are solving and explain your computations. Unclear solutions will be considered wrong.

Exercise 1

Consider the utility function

$$u(x) = \log(a + x), \quad a \geq 0, \quad (1)$$

defined over positive amounts of money. Prove that $u(x)$ is increasing and concave. Compute its absolute risk aversion coefficient.

Assume that there are two investment opportunities. The first is a risk-less security, giving a gross return of 1.2 monetary units in period 2 for each monetary unit invested in period 1. The second is a risky asset which pays, for each monetary unit invested in period 1, a yield of 3.6 monetary units in period 2 with probability $2/3$, and zero with probability $1/3$.

Considering a risk averse investor whose preferences are described by (1) with an initial endowment equal to 10 monetary units:

- assume $a = 1$ and compute the Expected Utility of investing 5 monetary units in the risky asset;
- assume $a = 0$ and compute the Expected Utility of splitting the endowment among the two securities, that is investing 5 monetary units in the first asset and 10 in the second;
- find the share of the initial endowment optimally invested in the risky asset for $a = 0$ and $a = 6$.

Exercise 2

Consider the following function

$$u(x) = 12x - 3x^2. \quad (2)$$

Prove that this function is a risk-averse utility function for monetary outcomes in $[0, 2]$.

Consider a lottery whose payoff is a real number randomly extracted with uniform probability in the closed interval $[0, 1]$. Compute the certainty equivalent of this lottery under the utility in (2). *Hint: Only one result is acceptable.*