Exam of "Decision under uncertainty", Pisa 07/06/2011

Solve the following two exercises, motivating your answers. Please notice that: you can keep with you the notes and the textbook of the course, but no other books are allowed; you can use a pocket/scientific calculator, but computers or other advanced devices able to perform symbolic computation are not allowed; at the end of the exam, you are required to deliver all the paper (draft included) to the teacher.

Exercise 1

Consider the following one-parameter family of utility functions defined over non negative amounts of money

$$u_a(x) = \sqrt{a + x} \quad \text{with} \quad a \geq 0 \,. \tag{1}$$

Prove that $u_a(x)$ is increasing and concave. What is the most risk averse utility function between $u_{10}(x)$ and $u_{20}(x)$?

Consider the following two lotteries

- lottery L_1 pays a positive outcome A with probability $\pi \in (0, 1)$ and zero otherwise;
- lottery L_2 pays a positive outcome $4A$ with probability $\pi/2$ and zero otherwise.

Find the value of a for which a decision maker using (1) is indifferent between the two. Prove that a lottery paying $2A$ with probability $\pi/2$ and zero otherwise is never preferred to L_1 under (1).

Consider a lottery L_3 whose payoff is a real number randomly extracted with uniform probability in the closed interval $[0, 3]$. This means that any $x \in [0, 3]$ has the same probability to be extracted. Compute the certainty equivalent of L_3 under u_1. *Hint: The probability distribution associated with the lottery L_3 is $F(x) = x/3$.*

Exercise 2

Consider the following three lotteries:

- lottery L_1 pays 10 with probability .5 and zero otherwise;
- Lottery L_2 pays 8 with probability .6 and zero otherwise;
- Lottery L_3 pays 12 with probability .4 and zero otherwise.

Assume that $L_2 \succ L_1$, that is the decision maker prefers lottery L_2 to L_1. Then consider these two new lotteries:

- lottery L'_1 pays 12 with probability .2, 10 with probability .25 and zero otherwise;
- lottery L'_2 pays 12 with probability .2, 8 with probability .3 and zero otherwise.

Prove that if the decision maker's preferences are consistent with Expected Utility Theory, than $L'_2 \succ L'_1$, that is lottery L'_2 is preferred to lottery L'_1. Notice that the outcomes of different lotteries are assumed independent. *Hint: why did I provide you the definition of L_3?*