On the bosonic nature of business opportunities

Giulio Bottazzi

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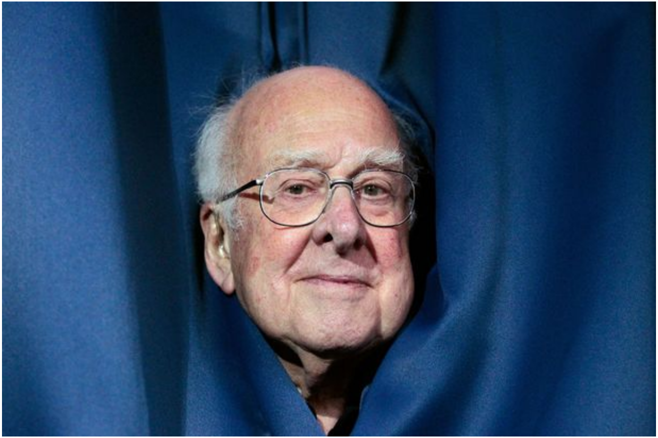
Who's this guy?



Peter Higgs, Nobel Prize laureate for the theoretical prediction in 1969 of the Higgs' boson whose existence "recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS

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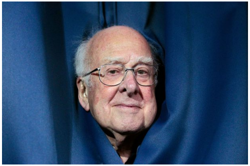

Peter Higgs, Nobel Prize laureate for the theoretical prediction in 1969 of the Higgs’ boson whose existence “recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN’s Large Hadron Collider"

Why Higgs decided to call his particle “boson”?

The origins of “bosons”

Actually he did not decide. He was this guy who decided, many years before. Paul A.M. Dirac. According to quantum theory sub-atomic particles are divided in two “groups”: Fermions and Bosons. The name “boson” was in honour of Satyendra Nath Bose

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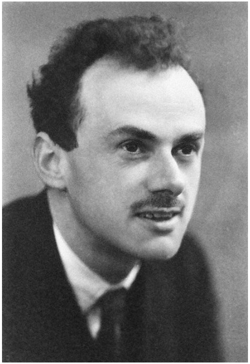

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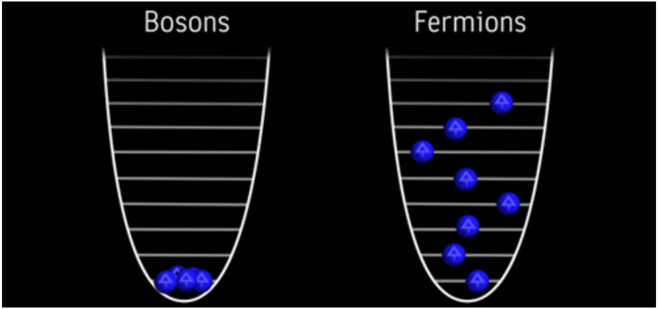
Bosonic and fermionic behavior

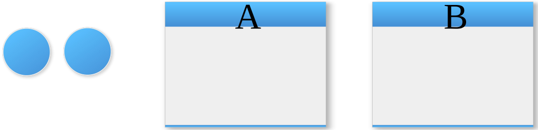
Bosons and fermions follow peculiar statistical rules when they distribute in the “energy levels”: Bose-Einstein and Fermi-Dirac statistics.



The difference of Bose-Einstein statistics with respect to classical (macroscopic) objects is better explained with an example.

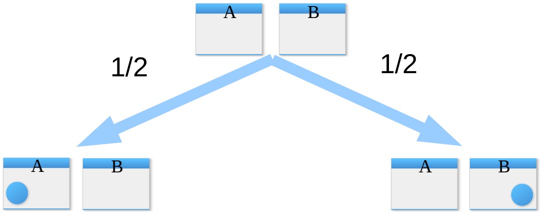
The simplest case

Two balls have to be assigned to two bins.



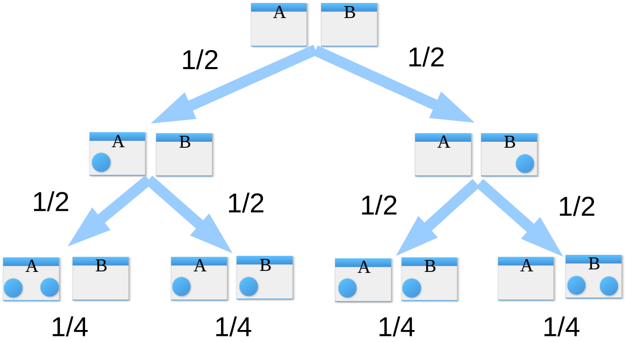
Assign the 1st ball

We assume that the probability for the two bins is the same



Assign the 2nd ball

Again the probability for the two bins is the same



Probability of different occupancies

Occupancy = way of distributing balls in bins. Classic balls: the even occupancy is the *most* probable. Bose-Einstein: All occupancies are equally probable.

Classical

P{ 

}= 1/4

P{ 

}= 1/2

P{ 

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	Classical	Bose-Einstein
P{  }=	1/4	1/3
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Distribution of the number of bins

Classical: the highest probability associated with the *mean* assignnment. Bose-Einstein: higher probability assigned to the "extreme" events.

	Classical	Bose-Einstein
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Many bins and balls

Let N be the number of bins and M the number of balls. When N and M becomes large: Bose-Einstein $\rightarrow$ Geometric and Binomial $\rightarrow$ Normal

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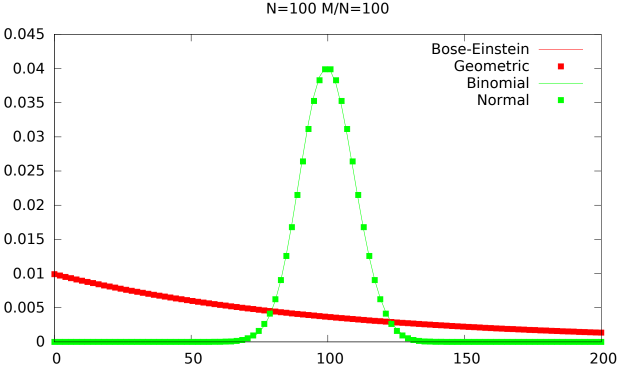

Pharma industry NCEs

New Chemical Entity (NCE): new molecules with novel therapeutical properties

The total number of NCEs introduced over the period 1975-1994 is 154.

In G.Bottazzi, G.Dosi, M.Lippi, F.Pammolli and M.Riccaboni, International Journal of Industrial Organization 2001 we analized the 150 top worldwide pharma firms.

How are the NCEs distributed among these top firms?

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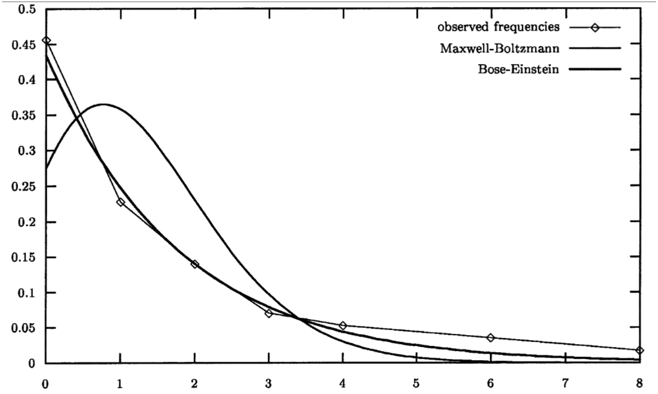
The distribution of NCEs

In this case it is $N = 150$ and $M = 154$.



The distribution of NCEs

The “mainly bosonic” nature of NCEs was easily established.

But we lack a list of “business opportunities” in the different sectors.

Thus to asses their bosonic nature we have to revert to an indirect proof.

It turns out that the shape of the firm growth rate distribution provides such proof. . . but we need a little more theory!

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Firm growth rate

Let $S(t)$ be the size of the firm at time t and $s(t) = \log S(t)$. Observed growth is the cumulative effect of diverse independent shocks

$$g(t) = s(t+1) - s(t) = \epsilon_1(t) + \epsilon_2(t) + \ldots = \sum_{j=1}^{N} \epsilon_j(t)$$

Let μ_ϵ and σ_ϵ^2 be the mean and variance of the shocks, then

$$\mathrm{E}[g] = N\mu_\epsilon \quad \mathrm{V}[g] = N\sigma_\epsilon^2 \ .$$

If N becomes large and $\mu_\epsilon, \sigma_\epsilon^2 \sim 1/N$ we have many micro-shocks. What we expect to observe as the distribution of g?

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Firm growth rates

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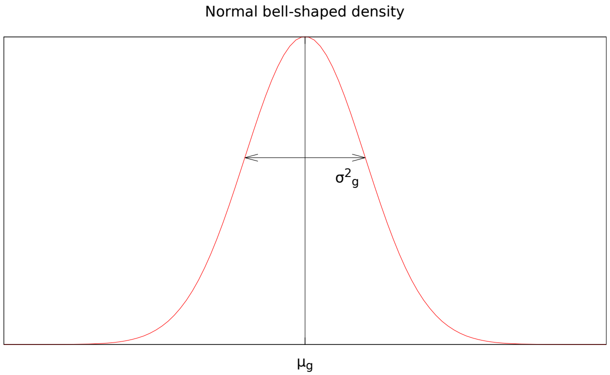

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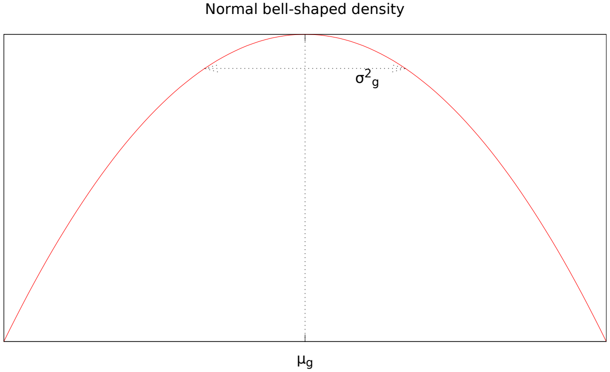

Random opportunities assignment

The previous model had no competition in it: each firm takes a large but FIXED number of opportunities.

Assume instead that micro-schocks arise from opportunities that are distributed among firms, like microscopic particles among energy levels.

Thus N becomes a random variable $\mathbf{N}$. Observed growth as the cumulative effect of diverse shocks

$$g(t) = s(t+1) - s(t) = \epsilon_1(t) + \epsilon_2(t) + \ldots = \sum_{j=1}^{\mathbf{N}} \epsilon_j(t) \ . \tag{1}$$

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Many firms and opportunities

With $N = \mathrm{E}[\mathbf{N}]$ it is again

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What happens if $N \to +\infty$, and $\mu_\epsilon, \sigma_\epsilon^2 \sim 1/N$, that is if we have many micro-shocks? What we expect to observe as the distribution of g?

It depends on the assignment procedure.

If the opportunities are distributed as classical particles, we are back to the Normal distribution.

If opportunities are distributed as "bosons" then in G.Bottazzi and A.Secchi, Rand Journal of Economics, 37, 2006 we showed that …

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Laplace density

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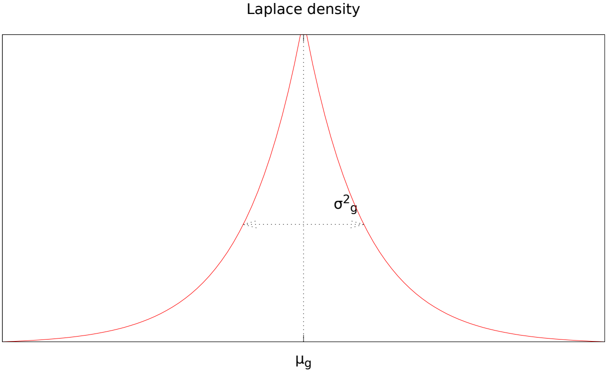

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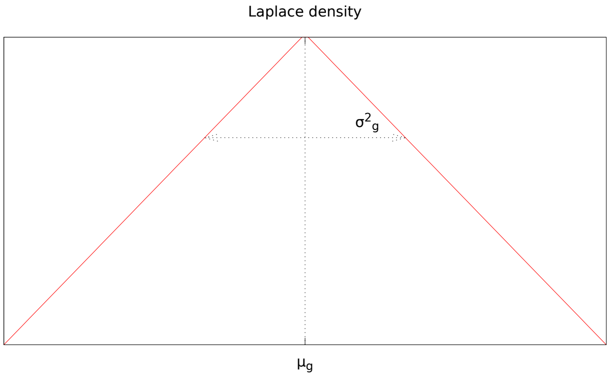
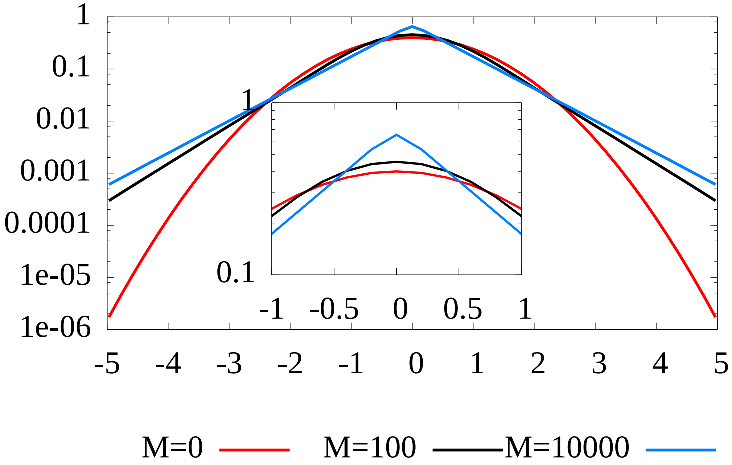
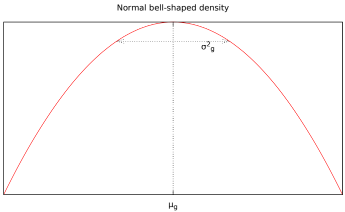


Simulation Results for $N = 100$

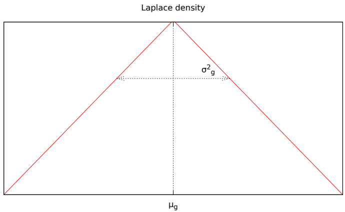


from G.Bottazzi and A.Secchi *Explaining the Distribution of Firms Growth Rates*, Rand Journal of Economics, 37, 2006

Alternative models



Classical opportunities: normal distribution



Bosonic opportunities: Laplace distribution

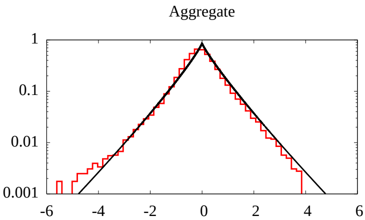
Data - U.S.

COMPUSTAT U.S. publicly traded firms in the Manufacturing Industry (SIC code ranges between 2000-3999) in the time window 1982-2001.

analysis performed in

G.Bottazzi and A.Secchi Review of Industrial Organization, vol. 23, pp. 217-232, 2003

Empirical Growth Rates Densities - U.S.



Two digits sectors

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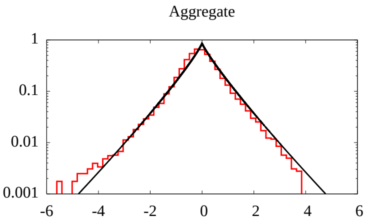

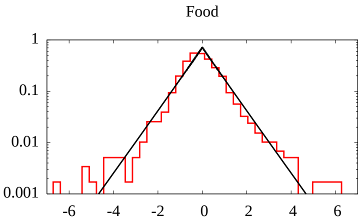

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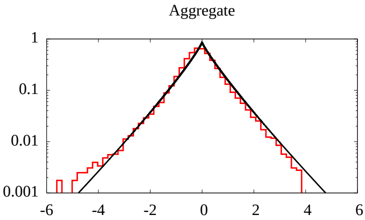

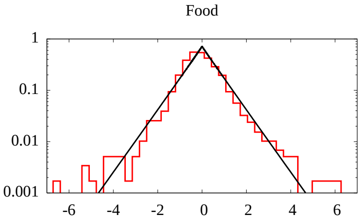

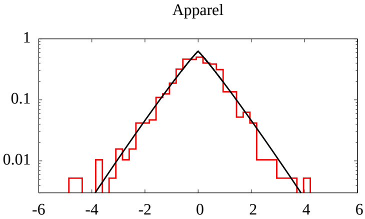

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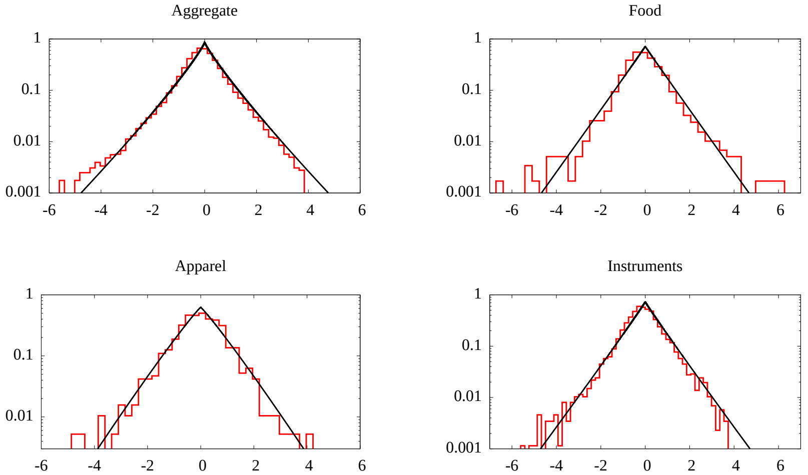

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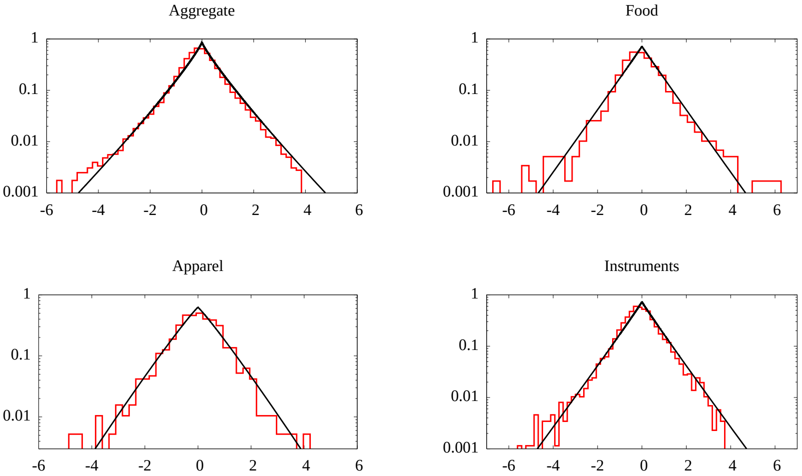


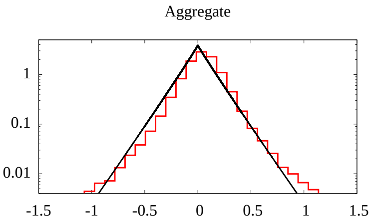
Two digits sectors

Data - Italy

MICRO.1 Developed by the Italian Statistical Office(ISTAT). Ten thousands firms with 20 or more employees in 97 sectors (3-digit ATECO) in the time window 1989-1996. We use 55 sectors with more than 44 firms.

analysis presented in
G.Bottazzi, E.Cefis and G.Dosi *Industrial and Corporate Change* vol. 11, 2002;
G.Bottazzi, E.Cefis, G.Dosi and A.Secchi *Small Business Economics* 29, pp. 137-159, 2007

Empirical Growth Rates Densities - ITA



Three digits sectors

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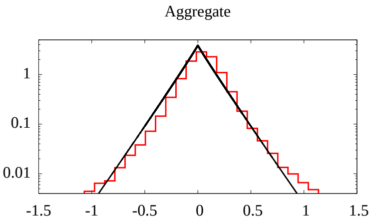
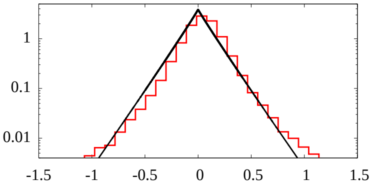


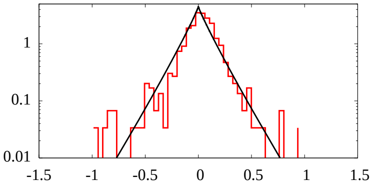
Three digits sectors

Empirical Growth Rates Densities - ITA

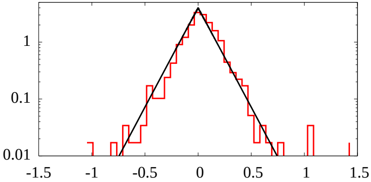
Aggregate



Pharmaceuticals



Cutlery, tools and general hardware



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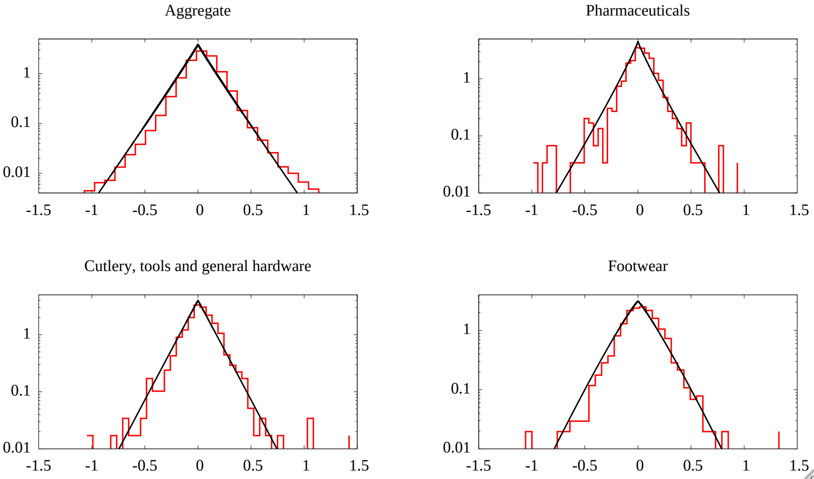

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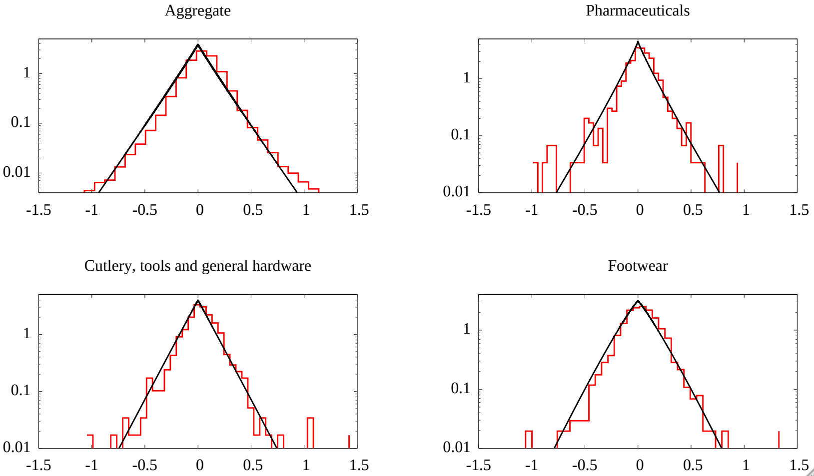


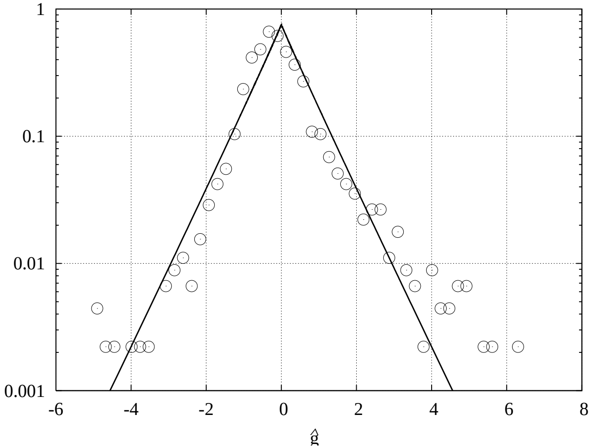
Three digits sectors

Data - worldwide pharma industry

PHID Developed by the CERM research institute. Sales figures for top pharmaceutical firms in United States, United Kingdom, France, Germany, Spain, Italy and Canada for the years 1987-97.

analysis presented in
G.Bottazzi and A.Secchi *Review of Industrial Organization*, 26, 2005

Empirical Growth Rates Densities - Pharma



A more general result

Theorem

Let $\mathbf{g}(\lambda, \mu, \sigma) = \sum_{\mathbf{j=1}}^{\mathbf{N}} \epsilon_{\mathbf{j}}(\mathbf{t})$ *with* ϵ *i.i.d distrbuted according to a common distribution with mean* μ *and variance* σ^2*, and* $\mathbf{N}$ *distributed according to a distribution* h *of mean* λ*. Assume that there is an* n' *such that for any* $n > n'$ *it is*

$$\lim_{\lambda \to +\infty} \lambda \sup_{n > n'} \{h(n) - G(n)\} = 0$$

where $G(n)$ *is the geometric distribution with mean* λ*, then*

$$\lim_{\lambda \to +\infty} \mathbf{g}(\lambda, \mu/\lambda, \sigma/\lambda) \sim \textit{Laplace}$$

In other words . . .

It in not necessary to have a Bose-Einstain statistics, but any way to distribute opportunities which leads, in the limit of a large number of opportunities, to a geometric marginal distribution will work!

The meansing of the Geometric distribution is

$$\mathrm{Prob}\{\mathbf{N} = \mathbf{n} + \mathbf{1}\} = \mathrm{cosntant} \times \mathrm{Prob}\{\mathbf{N} = \mathbf{n}\}$$

that is the probability to get one more opportunity is proportional to the number of opportunities already got.

This is the Gibrat's Law of Proportionate Effect or "competition among objects whose *market success*...[is] cumulative or self-reinforcing" (B.W. Arthur)

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Conclusions

The business opportunities follow the Bose-Einstein statistics.

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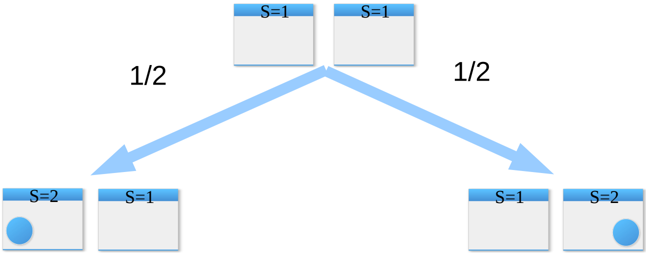
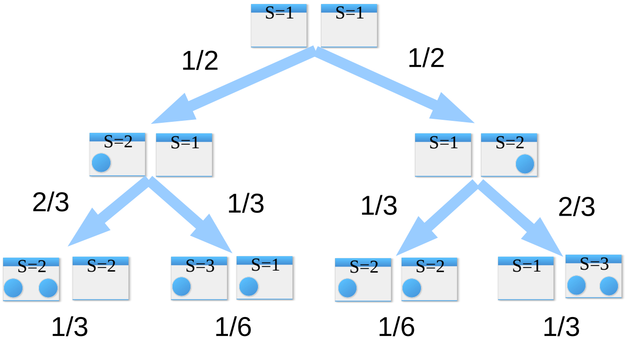
Self-reinforcing in the number of bins

As suggested in Y. Ijiri and H. Simon *Proc. Nat. Acad. Sci. USA*, 1975 assign bins proportionally to bin's "size". Initially attach to each bin the same size $S = 1$.



Self-reinforcing in the number of bins

Increase the size with the number of balls.



Diversification patterns

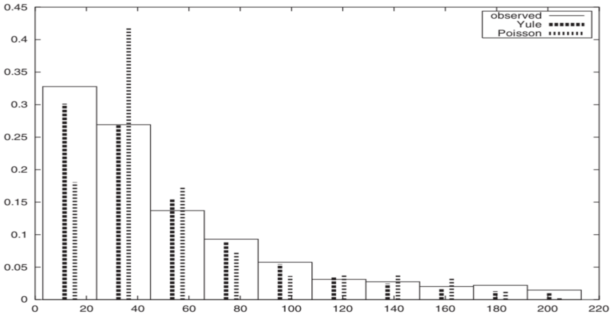


Figure 11 The binned probability density for the number of sub-market computed directly from the data and theoretically predicted by the Yule process. The theoretical distribution is characterized by $\lambda = 0.35$, $n_0 = 5$ and $s_0 = -12$. For comparison, a fit of a Poisson model with a non-linear intensity function $\Lambda(s) = s^{\lambda}$ is also shown.

from G.Bottazzi and A.Secchi *Industrial and Corporate Change*, 2006