Modeling firms locational choice

Giulio Bottazzi

DIMETIC School
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Scuola Superiore Sant'Anna
di Studi Universitari e di Perfezionamento

Introduction

Agglomeration derive from some form of externality.

Drivers of agglomeration can be of two types: pecuniary and non-pecuniary.

Pecuniary: local final demand, intermediate market for input goods

Non-pecuniary: technological spillover, local knowledge (tacit), institutional setting

not sure: labor market (skilled labor can be "generic" skill OR specific "skill")

Compare the two, empirically and theoretically

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Outline

1 The relevance of externalities
- The Model
- Simulations
- Analytical result

2 Detecting technological spillover

3 Dynamics in Economic Geography
- Introduction
- The static model
  - No technological spillover
  - Localized technological spillover
- The dynamic model
  - Agglomeration and equidistribution
  - Comparative dynamics
- Conclusions

Location selection under dynamic externalities

Modeling industrial evolution in geographical space, JEG 7 (2007) pp. 651-672

N firms have to select among L locations.

Time is discrete time: at each time step a firm is relocated (or entry/exit).

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Firm decision

$$\text{Profit of firm } i \text{ to locate in } l = a_l + b_l n_l + \epsilon_{i,l}$$

n_l the number of firms already there, $\epsilon_{i,l}$ idiosyncratic component.

Probabilistic discrete choice model (Thurstone (1927), Luce (1959))

$$\text{Prob firm } i \text{ select location } l = a_l + b_l n_l$$

Occupancy vector $\boldsymbol{n}_t = (n_{1,t}, ..., n_{L,t})$ describes the state of the economy.

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Intrinsic Attractiveness - Economic Interpretation

Intrinsic attractiveness a: perceived gains that a firm would obtain by choosing l net of any agglomeration effects.

1. sheer geographical aspects (a harbor or a river) including sticky man-made factors
2. enabling conditions and "catalyzers" like locally available skilled labor and knowledge spillover from thereby universities
3. externalities (suppliers or customers availability) that are endogenous to the location as a whole but exogenous to any particular "small" sector of activity

Agglomeration Economies - Economic Interpretation

Strength of agglomeration economies b: measures the amount by which the advantages obtained by locating in l increases as a function of the number of firms already located there

1. technological externalities
2. sharing of fixed costs
3. local spin-off (entry/exit process)

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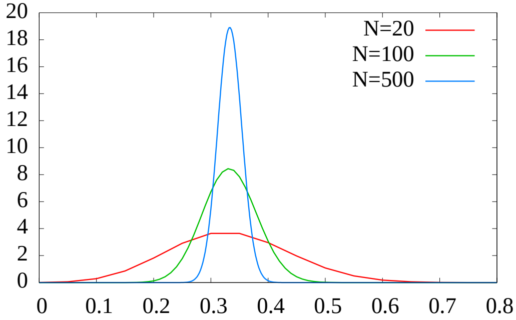
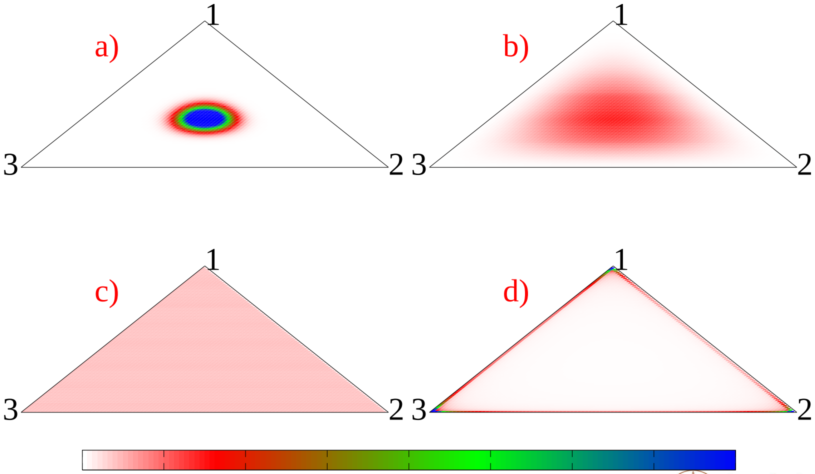
2 locations and No Agglomeration Feedbacks

Location 1 is occupied, on average, by a number of firms $\sim a_1/(a_1 + a_2)$.



Probability density of the fraction of firms in location 1 for $b_1 = b_2 = 0$ and $a_1 = 1$ and $a_2 = 2$.

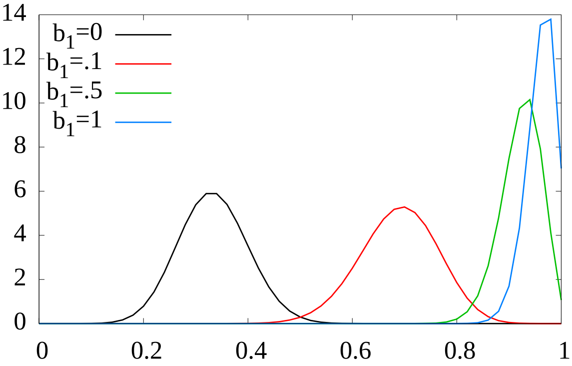
3 Locations with equal Agglomeration Feedbacks



The 3 locations have the same intrinsic attractiveness $a = 1$ but

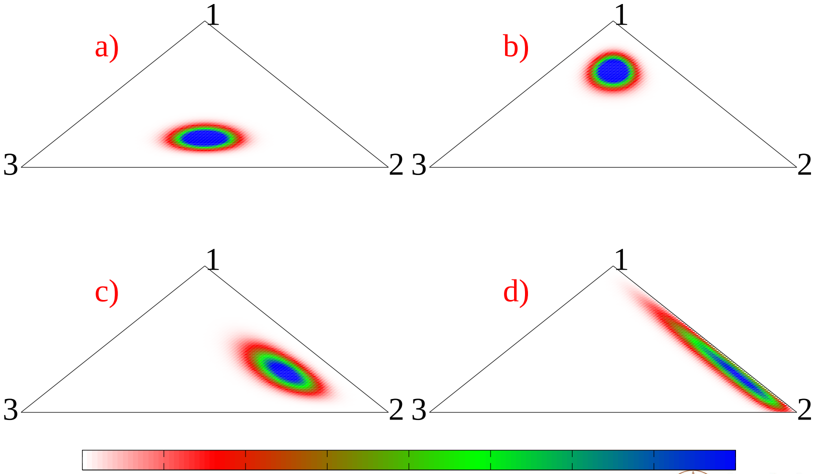
a) $b_1 = b_2 = b_3 = 0$ b) $b_1 = b_2 = b_3 = 0.3$

2 locations with diverse Agglomeration Feedbacks



Probability density of the fraction of firms in location 1 for different values of b_1 with $a_1 = 1$, $a_2 = 2$ and $b_2 = 0$.

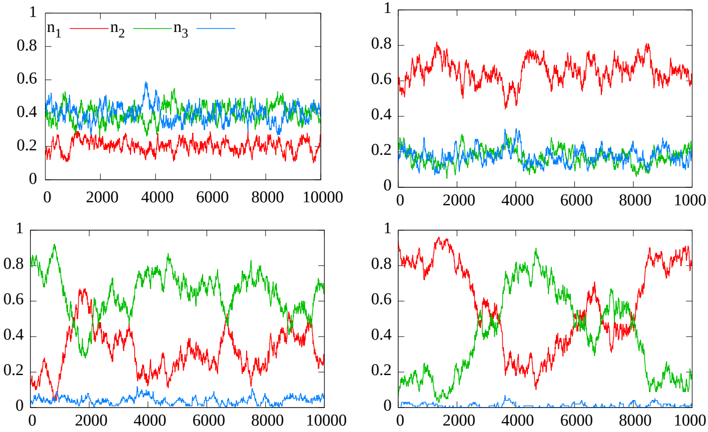
3 Locations with Diverse Agglomeration Feedbacks



The 3 locations have intrinsic attractiveness $a_1 = 1$ and $a_2 = a_3 = 2$

a) $b_1 = 0$ $b_2 = 0$ $b_3 = 0$ b) $b_1 = 0.1$ $b_2 = b_3 = 0$

Temporal Dynamics of Firms Shares



The 3 locations have intrinsic attractiveness $a_1 = 1$ and $a_2 = a_3 = 2$

a) $b_1 = 0$ $b_2 = 0$ $b_3 = 0$ b) $b_1 = 0.1$ $b_2 = b_3 = 0$

c) $b_1 = b_2 = 0.5$ $b_3 = 0$ d) $b_1 = b_2 = 2$ $b_3 = 0$

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The Polya Distribution

This analytical framework admits a unique stationary distribution $\pi(\boldsymbol{n};\boldsymbol{a},\boldsymbol{b})$.

Assuming $b_l = b \; \forall l$ the probability $\pi(n;a,b)$ of finding n firms in a location with attractiveness a is

$$\pi(n;a,n) = \binom{N}{n} \frac{\Gamma(A/b)}{\Gamma(A/b+N)} \frac{\Gamma(a/b+n)}{\Gamma(a/b)} \frac{\Gamma((A-a)/b+N-n)}{\Gamma((A-a)/b)} .$$

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The Polyit model

Imagine to have

- A set of location $1, \ldots, L$.
- A set of location-specific regressors X_l.
- The number of economic unit n_l in each location.

Consider the specification $\quad p_l(n, b) = X_l' \beta$.

Using the observed occupancy n_l, maximize the likelihood of the Polya distribution $L = \log \pi \left(\boldsymbol{n}; X_l' \beta, b\right)$ to obtain $(\hat{\beta}, \hat{b})$.

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Sectoral analysis

Sectoral and geographical specificities in the spatial structure of economic activities SCED 19 (2008) 189-202

"Census of Manufacturers and Services" (ISTAT) BU and employees are classified with respect to 784 geographical locations and ISIC industrial sectors.

$$n_{j,l} = \#\text{ of firms or employees in location } l \text{ sector } j$$

For each sector j consider the specification

$$p_{j,l}(n; b, \beta) = \beta\, n_{-j,l}$$

β captures "urbanization" effects. $(\hat{b}_j, \hat{\beta}_j)$ for each sector.

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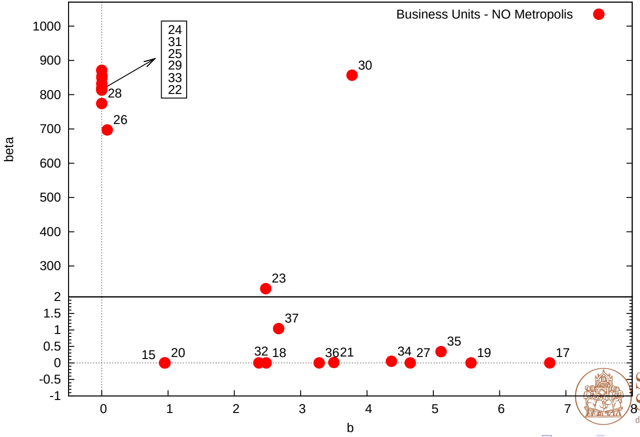
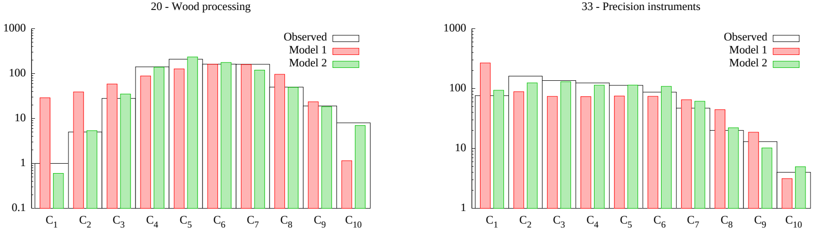
n = number of BUs

Metropolis Excluded



Goodness of fit



Occupancy class frequencies computed on observed data (white bars) and estimated using Model 1 (red bars) and Model 2 (green bars).

Summarizing

Dynamic micro-economic model with choice under uncertainty: probabilistic notion of equilibrium.

We used it to:

- disentangle location-specific and sector-specific forces of agglomeration.
- assess the relevance of sector-specific agglomeration economies
- produce empirically testable hypothesis on the whole spatial distribution of economic activities

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Overview

Extending New Economic Geography (NEG) analysis including non pecuniary externality inside a tractable evolutionary model of firms location.

Benchmark model (as Krugman, 1991) with increasing return and pecuniary externalities + immobile workers and mobile capital (Forlsid and Ottaviano 2003 use “skilled labour”).

Modified in three ways (see e.g. Frenken and Boschma, 2007):

1. Direct firms interaction via technological externalities
2. Explicit time dimension
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NEG settings

- 2 locations.
- I households per location, global consumers and local workers, demand for a bundle of manufacturing goods and one agricultural good.
- $n_1 + n_2 = N$ firms, single input (labour) production with increasing return
- Transportation cost τ as iceberg cost.

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Household and Firms

Household maximize CES utility for a demand

$$\frac{\partial \log c}{\partial \log p} = -\sigma + (\ldots)$$

Firm in l_i faces cost function

$$v(y) = (\beta y + \alpha_{l_i})\ w_{l_i}\ , \qquad y = \text{output} \quad w = \text{wages}$$

β constant and α location specific.

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Market structure

Agricultural sector is global (zero transport cost): wages are equal in both locations and set to 1.

Assuming monopolistic competition for firms, equality of wages imply

$$p = \sigma/(\sigma - 1)\beta \ .$$

No α_{l_i}: same price in both locations.

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Economic (location-by-location) equilibrium

Determine prices and quantities given n_1 and and n_2:

- Consumer budget constraint and CES function determine demanded quantities in both locations.
- Equating global demand and supply determines firms production.
- Output price and cost structure set the level of profits in the two locations

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Short-run profits

Set $x = n_1/N$. Profits per location read

$$\begin{cases} \pi_1(x) = \dfrac{I}{N\sigma}\left(\dfrac{1}{x+(1-x)\tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{x\tau^{\sigma-1}+(1-x)}\right) - \alpha_1\,, \\ \pi_2(x) = \dfrac{I}{N\sigma}\left(\dfrac{1}{x\tau^{\sigma-1}+(1-x)} + \dfrac{\tau^{\sigma-1}}{x+(1-x)\tau^{\sigma-1}}\right) - \alpha_2\,. \end{cases}$$

Endowment $\Uparrow$ Local Dem. $\Uparrow$ Foreign Dem. $\Uparrow$ Costs $\Uparrow$

Traditional model

Assumption

Fixed costs are constant across sectors and locations, $\alpha_1 = \alpha_2 = \alpha$.

From the equation above

$$x\pi_1(x) + (1-x)\pi_2(x) = \frac{2I}{N\sigma} - \alpha$$

Long run equilibrium gives

$$N \to \frac{2I}{\sigma\,\alpha}$$

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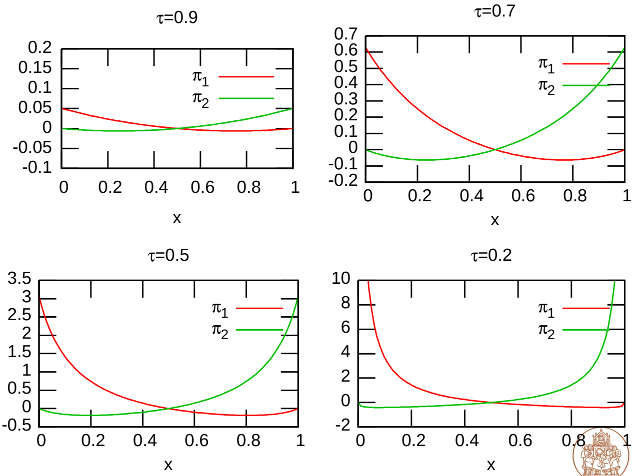

Geographical equilibrium:

Theorem

There always exists only one symmetric geographical equilibria for $x^* = 0.5$. *The border distribution* $x_1^* = 1$ *and* $x_0^* = 0$ *are never equilibria.*

Non-pecuniary externalities

Assumption

Fixed costs are locally shared

$$\alpha_l = \frac{\alpha N}{2 n_l}$$

Fixed costs are a function of firms concentration: knowledge spillover, access to specific skilled labor pool, use of service or infrastructure.

Same long run equilibrium

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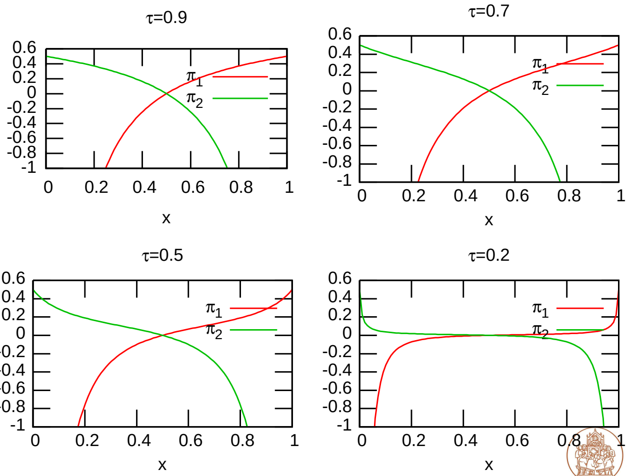

Geographical equilibrium

Theorem

There always exists two, and only two, geographical equilibria given by the border distribution $x_1^* = 1$ *and* $x_0^* = 0$*. In particular, the unique distribution where profits are equal,* $x^* = 0.5$*, is never an equilibrium.*

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Entry-Exit process

Out of equilibrium process: one firm at a time is randomly selected (uniformly) and updates its location choice.

Firm i maximizes "perceived" profit

$$Payoff_i = \pi_{l_i} + \varepsilon_{i,l_i} \, .$$

Choice is probabilistic with

$$p_l = \frac{e^{\pi_l}}{e^{\pi_1} + e^{\pi_2}} \, , \quad l \in \{1, 2\} \, . \qquad (1)$$

but π_i depends on choice of all other firms.

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Firm i maximizes "perceived" profit

$$Payoff_i = \pi_{l_i} + \varepsilon_{i,l_i} .$$

Choice is probabilistic with

$$p_l = \frac{e^{\pi_l}}{e^{\pi_1} + e^{\pi_2}} , \quad l \in \{1,2\} . \tag{1}$$

but π_i depends on choice of all other firms.

Towards a dynamic geographical equilibrium

With p_l linear in x_l the equilibrium distribution can be computed.

Theorem

Denote linearized profits around $x^* = 0.5$ as c_l, and the number of firms in location l as n_l. They read

$$c_l = a + bn_l\,, \quad l = 1, 2\,,$$

where intrinsic profit a and marginal profit b are

$$a = 1 - \frac{4\alpha\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\,, \qquad b = \frac{4\alpha^2\sigma\tau^{\sigma-1}}{I(1+\tau^{\sigma-1})^2}\,.$$

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Geographical equilibrium distribution

Bottazzi et al. (2007), Bottazzi and Secchi (2007)

Theorem

The model with linearized profits admits a unique stationary distribution

$$\pi(\boldsymbol{n}) = \frac{N!C(N,a,b)}{Z(N,a,b)} \prod_{l=1}^{2} \frac{1}{n_l!} \vartheta_{n_l}(a,b) \,,$$

where

$$C(N,a,b) = 2a + \left(1 - \frac{1}{N}\right) bN \,, \tag{2}$$

$$\vartheta_n(a,b) = \begin{cases} \prod_{h=1}^{n} [a + b(h-1)] & n > 0 \\ 1 & n = 0 \end{cases} \tag{3}$$

and $Z(N, a, b)$ is a normalization factor which depends only on the

Recovering different phases

The push toward symmetry of pecuniary externalities increases (decreases) with transportation cost (τ).

Theorem

When the marginal profit is bigger than the intrinsic profit, $b > a$, the equilibrium distribution of the entry-exit process is bimodal with modes in $x = 0$ and $x = 1$, when $b < a$ the equilibrium distribution is unimodal with mode in $x = 0.5$, and when $a = b$ the equilibrium distribution is uniform.

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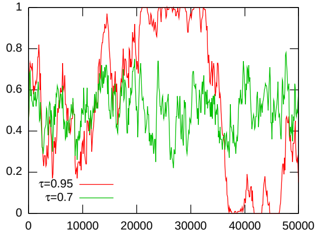
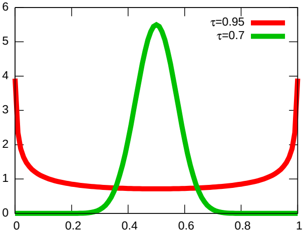
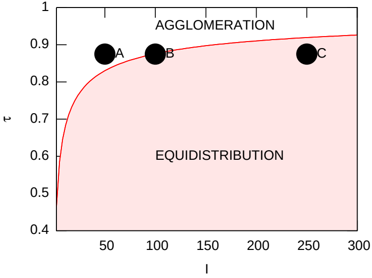
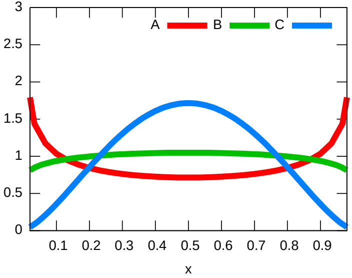
Simulations and stationary distributions



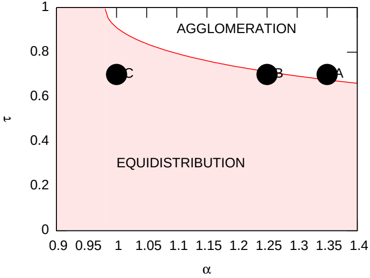


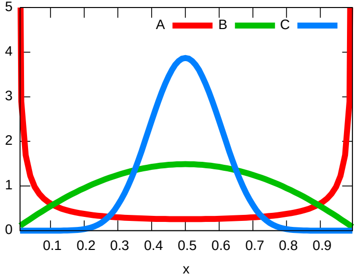
Comparative dynamics: Number of households





Comparative dynamics: Fixed costs





Outline

1. The relevance of externalities
   - The Model
   - Simulations
   - Analytical result
2. Detecting technological spillover
3. Dynamics in Economic Geography
   - Introduction
   - The static model
     - No technological spillover
     - Localized technological spillover
   - The dynamic model
     - Agglomeration and equidistribution
     - Comparative dynamics
   - Conclusions

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- Technological externality strong (too strong?) source of agglomeration
- More plausible in an heterogeneous framework: idiosyncrasies reduce core-periphery likelihood.
- Agglomeration is a meta-stable phenomenon
- Future work:
  - Robustness of these results to modeling assumptions
  - Micro-foundation of technological externality
  - Calibration/estimation of the model with real data

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