Financial Constraints and Firm Dynamics

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Outline



- Motivation
- Aim of the paper
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- 3 Analytical framework
- 4 Robustness checks

Firm size distribution, age and financial constraints Analytical framework Robustness checks Motivation Aim of the paper Data description

Introduction & data

- Motivation
- Aim of the paper
- Data description

2 Firm size distribution, age and financial constraints

- 3 Analytical framework
- 4 Robustness checks

Motivation Aim of the paper Data description

Available evidence on FCs and firms' dynamics

- structured / complex impact of FCs: FCs affect many dimensions of firms' decisions and evolution
 - investment/divestment decisions
 - decision to expand production or entering new markets
 - cash management
 - R&D policies ...
- Qualitative evidence on reaction to crises (Campello, Graham and Campbell, NBER2009) suggests heterogenous impact of FC:
 - "Pinioning effect": firms facing good opportunities tend to bypass attractive investment projects
 - "Loss reinforcing" effect: firms facing poor growth opportunities display higher propensity to sell off productive assets to generate funds, further deteriorating growth prospects

Motivation Aim of the paper Data description

Empirical background: regression analyses

Long tradition of studies on the effects of FC on firms' decisions and evolution

- FC is a significant determinant of firms' investment decisions: Fazzari, Hubbard and Petersen, Brooking Papers(1988).
- FC has impact on firm growth: Deveraux and Schiantarelli, NBER WP(1990); Becchetti and Trovato, SBE(2002); Desai, Gompers and Lerner, NBER WP(2003).

Traditional approach: augmented Gibrat's regression

$$s_t - s_{t-1} = c + \lambda \ s_{t-1} + \beta \ \text{FC-Proxy} + \epsilon_t$$

Limitation 1: it captures central effect of FCs on growth

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Empirical background: distributional analyses

Cabral and Mata, AER(2003)

- FSD of a cohort is skewed at time of birth and gradually evolve toward more symmetric distribution
- The evolution of FSD is determined by firms ceasing to be FC
- Angelini and Generale, AER(2008)
 - negative relation between FC and firm size: FC firms are smaller and their FSD is more skewed; FC matter only for a small group of firms(~5.8% of their database)
 - FC is not the main determinant of the FSD evolution

Limitation 2: They ignore the relation between short-run and long-run effects of FS

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Assume I(1). If FCs generate a shift in expected growth rates, we would observe a shift in FSD.

Bo-Se-Ta Financial Constraints and Firm Dynamics

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What you (hopefully) get by reading this paper

There are remarkable effects of FCs on firms' growth that are overlooked in a standard regression analysis framework. These effects have a sound economic interpretation.

To identify them we

move beyond the traditional regression approach and account for

- non-normal growth shocks
- non linear heteroskedastic effects

focus on distributional properties of growth allowing FCs to affect differently

- growing and not-growing firms (check for induced asymmetries)
- fast growing firms with respect to slow growing one (look at the tails of the distribution)

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DATA SET: sample from the Italian CADS - Company Account Data Service, including annual reports for approximately 165,000 *limited liability* firms, active in Manufacturing over the period 2000-2003. Cover 50% of total employment, 60% of Valued Added and around 7% in terms of number of firms.

DATA PROVIDER: CEBI founded as an agency of the Bank of Italy and the Italian Banking Association in the early 80's with the institutional task of monitoring risk exposure of Italian banking system. It is nowadays a private company owned by major Italian banks.

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Cleaning and data selection

Few anomalous observations are removed. Details.

Variable considered are SIZE (Total sales), ASSETS (Net tangible assets), PROFIT (Gross operating margin) and CEBI rating index:

- Rating score in the range 1-9 but not cardinal
- built using multivariate discriminant analysis: firms expected ability to pay back their loans
- higher score firms are smaller, more leveraged and they pay higher interest rates. (Panetta, Schivardi and Schum, 2009 JMCB).

FC proxy built using CEBI rating

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Using rating as FC proxy

- Similarly to recently developed multivariate indeces of FCs (Cleary, JF 1999; Lamont, Polk and Saá-Requejo, RFS 2001), credit ratings meet 3 crucial conditions:
 - provide a multidimensional assessment of firms' financial position
 - allow to avoid a simple binary categorization into constrained vs. not constrained firms
 - vary over time.
- 2 Alternative in the literature are survey based measures:
 - suffer from mis-perception and/or self-selection biases
 - capture the opinion of the "credit seekers" on "credit suppliers", while the opposite direction seems more relevant
- S CEBI rating vs. other ratings:
 - available for ALL firms in the data
 - due to institutional role of CeBi, we know banks rely on CeBi ratings in granting and pricing credit to firms

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FC classification

Three FC classes:

- Non Financially Constrained (NFC), rating in 1-4.
- Mildly Financially Constrained (MFC), rating in 5-7.
- Highly Financially Constrained (HFC), rating in 8-9.

Assignment based on the rating of the previous year (we also explored other possibilities).

Data pooled together after checking results are stationary.

FC classes are weakly related with economic variables but they affect probability of financial distress.

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Rating and economic variables 1

from Productivity, Profitability and Financial Performance by Bottazzi, Tamagni and Secchi, ICC 2008



Figure 2 Empirical density of ROS in 2002 for the manufacturing (A) and service (B) industry.



Figure 4 Empirical density of labor productivity differentials in 2002 for the manufacturing (A) and service (B) industry. Labor productivity is defined as VA/L.

Image: A matrix and a matrix

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Figure 6 Kernel estimates of the conditional expectation of output: Low-Risk firms in service (A) and in manufacturing (B)—2002. Sectoral plane via OLS.



Figure 7 Kernel estimates of the conditional expectation of output: Mid-Risk firms in service (A) and in manufacturing (B)—2002. Sectoral plane via OLS.



Figure 8 Kernel estimates of the conditional expectation of output: High-Risk firms in service (A) and in manufacturing (B)-2002. Sectoral plane via OLS.

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Rating and firm default

from Financial and Economic Determinants of Firm Default by Bottazzi, Grazzi, Secchi and Tamagni, JEE 2010

	Bootstrap probit with credit ratings-estimates by year							
	Rating only				Rating, financial and economic			
	(1) 1999	(2) 2000	(3) 2001	(4) 2002	(5) 1999	(6) 2000	(7) 2001	(8) 2002
Panel A: estimates IE/S LEV FD/S In SIZE PROD PROF GROWTH CONSTANT	-0.3694*	-0.4770*	-0.3720*	-0.1628*	0.0039* -0.0001 0.0044 0.0061* -0.0111* -0.0004 0.0054* -0.4999*	0.0041* -0.0013 0.0059 0.0072* -0.0075* -0.0015 -0.0000 -0.6582*	0.0049 -0.0015 0.0026 0.0090* -0.0059* -0.0021 -0.0038* -0.4960*	0.0088* 0.0019 0.0086* -0.0011 -0.0037* -0.0036* -0.0036*
LOW MID	-0.0301* 0.0198*	-0.0076 0.0614*	-0.0323* 0.0414*	-0.1206* -0.0072	0.0001 0.0392*	0.0186* 0.1134*	-0.0047 0.0639*	-0.0428* 0.0068
Panel B: model performance Brier score Threshold Type I error Type II error % Correct default % Correct non default	0.0328 0.0258 78.0000 576.1900 0.3906 0.8397	0.0327 0.0284 70.7850 616.3500 0.4757 0.8361	0.0322 0.0193 61.0700 709.3900 0.5606 0.8171	0.0319 0.0163 48.7550 633.7250 0.6278 0.8262	0.0325 0.0290 36.6350 1,248.9350 0.7138 0.6526	0.0326 0.0254 30.0450 1,334.5550 0.7774 0.6452	0.0320 0.0239 29.8300 1,230.2100 0.7854 0.6828	0.0316 0.0270 25.8250 901.1550 0.8029 0.7528
Panel C: comparisons of pred Threshold Type I error Type II error % Correct default % Correct non default	iction perform	ance against the	"rating only" m	odel of the same	year 0.0258 25.2550 1,612.8600 0.8027 0.5514	0.0284 39.5450 1,111.9150 0.7071 0.7044	0.0193 16.2850 1,722.5850 0.8828 0.5558	0.0163 11.8150 1,507.6900 0.9098 0.5865

Table 6 Probit estimates of default probabilities including credit ratings as modeled in Eqs. 10 and 11-results over 200 bootstrap replications

Variables are in z-scores. Panel A: Bootstrap means of marginal effects at the sample average of covariates. *Significant at 1% level. Panels B and C: Bootstrap means of model performance measures

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Relevance of the FC phenomenon

	Whole	Sample	Non Financia	lly Constrained	Mildly Financ	ially Constrained	Highly Financi	ially Constrained
Firm's age (years)	Number of firms	Size: mean (median)	Number of firms (% age class)	Size: mean (median)	Number of firms (% age class)	Size: mean (median)	Number of firms (% of age class)	Size: mean (median)
0-4	38,020	1.795	10,356	1.804	20,408	1.970	7,256	1.293
		(0.606)	(27.2)	(0.525)	(53.7)	(0.719)	(19.1)	(0.449)
5-10	52,150	3.369	18,269	4.115	27,862	3.248	6,019	1.666
		(0.860)	(35.0)	(0.844)	(53.4)	(0.995)	(11.5)	(0.439)
11-20	62,977	7.093	29,130	8.210	29,408	6.400	4,439	4.354
		(1.522)	(55.9)	(1.606)	(46.7)	(1.663)	(7.0)	(0.525)
21-30	35,579	10.139	18,966	11.147	15,080	9.544	1,533	3.520
		(2.674)	(53.3)	(2.719)	(42.4)	(2.921)	(4.3)	(0.696)
31-∞	20,645	25.917	11,374	26.600	8,213	22.157	1,058	47.760
		(4.516)	(55.1)	(4.919)	(39.8)	(4.764)	(5.1)	(1.345)
Total	209,371	7.577	88,095	9.614	100,971	6.386	20,305	4.662
		(1.301)	(42.1)	(1.548)	(48.2)	(1.371)	(9.7)	(0.494)

Table: FINANCIAL CONSTRAINTS BY AGE CLASSES

a Size as real sales, millions of euro.

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FSD and age





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FSD, age and financial constraints

NFC

MFC

HFC

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FSD evolution depends on the FC class:

- shift in the central location, smaller for the HFC class
- variance increases for NFC and MFC, less for HFC See stats
- right-tail tends to Gaussian for NFC and MFC, not for HFC Asymmetric Power Exponential(AEP)

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Augmented Gibrat's process

 $s_t = s_{t-1} + \epsilon_t$

Gibrat's benchmark is a good first approximation: seems good for NFC and MFC but not for HFC.

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Augmented Gibrat's process

$$s_t - s_{t-1} = c + \lambda \ s_{t-1} + \sigma(s_{t-1})\epsilon_t$$

Gibrat extended to include:

- Autoregressive coefficient λ
- Heteroskedastic correction σ
- Non-normality ϵ Asymmetric Laplace (ALAD regression)

Augmented Gibrat's process

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC} s_{t-1} + \sigma_{FC}(s_{t-1})\epsilon_t$$

Allow the coefficient to vary across FC classes: drop orthogonality assumption of residuals and partially solve omitted variable bias

The effect of λ

Different values of λ imply different evolutions of the FSD:



Given the barrier effect of FCs suggested by all previous analysis, we expect $\lambda < 0$, and thus NON Log-normal FSD, for more severely constrained firms

The need of $\sigma(s)$

Often reported negative relation between the variance of growth $g_{i,t} = s_{i,t+1} - s_{i,t}$ and size. Exponential fit (N.B.: does not depend on age)



Bo-Se-Ta Financial Constraints and Firm Dynamics

FCs and the distribution of growth rates

Qualitative evidence: "pinioning" and "loss reinforcing"

ASYMMETRIC DISTRIBUTIONAL EFFECT



Regression Analysis

We end up with Model 1

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC} s_{t-1} + \exp(\gamma_{FC} \cdot s_{t-1}) \epsilon_{FC,t}$$

estimated through ALAD.

	FC CLASS	Model 1
	NFC	
γ		-0.222*(0.001)
с		0.009*(0.001)
λ		-0.0007*(0.0003)

	MFC	
γ		-0.220*(0.001)
с		-0.011*(0.001)
λ		-0.0076*(0.0003)

HFC	
γ	-0.161*(0.002)
c	-0.013*(0.003)
λ	-0.030*(0.001)

* significantly different from zero at 1%. Sandwich errors.

Distribution of residuals by FC class: young firms



For younger firms, strong FCs :

- slim down the right tail of the distribution, i.e. shift of probability mass from the tail to the central part of the distribution
- do not seem to have an effect on the left half

Distribution of residuals by FC class: old firms

OLD NFC OLD HFC GROWTH RATES DENSIT TH MORE THAN 30 YEARS GROWTH RATES DENS WITH MORE THAN 30 YEARS 10 Empirical obs. ¹⁰ AEP fit Empirical obs. AEP fit AEP DISTRIBUTION PARAM AEP DISTRIBUTION PARAMI b.= 0.812 b= 0.750(0.015) b= 0.583(0.033) b.= 1.059 0.1 a=0.162(0.003) a = 0.134a = 0.3310a.= 0.373(0.021) m= 0.005(0.002) m= +0.078(0.008) 0.1 0.01 0.01 0.003 0.0001 0 .3 rescaled growth rates rescaled growth rates

For old firms, strong FCs:

lensity

- imply a very mild slim down of the right tail
- fatten up the left tail of the distribution, i.e. shift of probability mass from the central part to the tail of the distribution

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Attrition from sample censoring

Problem: higher exit rates for smaller firms + size relation n with FC \rightarrow bias in λ . Solution: split the sample in size classes (Eurostat def. in num. of emplo.) and re-estimate Model I.

Micro(0-9)	Small(10-49)	Medium(50-249)	Large(250+)
-0.2520(0.0014)	-0.1196(0.0045)	-0.1591(0.0052)	-0.1098(0.0104)
0.0056(0.0009)	0.0095(0.0010)	0.0009(0.0011)	-0.0058(0.0024)
-0.0115(0.0006)	-0.0114(0.0013)	-0.0064(0.0013)	-0.0024(0.0022)
-0.2407(0.0015)	-0.1707(0.0042)	-0.1704(0.0056)	-0.2307(0.0131)
-0.0171(0.0009)	-0.0117(0.0013)	-0.0142(0.0015)	-0.0290(0.0040)
-0.0220(0.0007)	-0.0308(0.0015)	-0.0102(0.0017)	-0.0057(0.0034)
-0.1715(0.0029)	-0.2219(0.0136)	-0.1995(0.0192)	-0.1339(0.0213)
-0.0067(0.0031)	-0.1264(0.0080)	-0.0366(0.0121)	-0.0916(0.0217)
-0.0736(0.0023)	-0.1272(0.0085)	-0.0630(0.0116)	-0.0141(0.0128)
	Micro(0-9) -0.2520(0.0014) 0.0056(0.0009) -0.0115(0.0006) -0.2407(0.0015) -0.0171(0.0009) -0.0220(0.0007) -0.1715(0.0029) -0.0067(0.0031) -0.0736(0.0023)	Micro(0-9) Small(10-49) -0.2520(0.0014) -0.1196(0.0045) 0.0056(0.0009) 0.0095(0.0010) -0.0115(0.0006) -0.0114(0.0013) -0.2407(0.0015) -0.1707(0.0042) -0.0171(0.0009) -0.0117(0.0013) -0.0220(0.0007) -0.308(0.0015) -0.1715(0.0029) -0.2219(0.0136) -0.0067(0.0031) -0.1264(0.0080) -0.0736(0.0023) -0.1272(0.0085)	Micro(0-9) Small(10-49) Medium(50-249) -0.2520(0.0014) -0.1196(0.0045) -0.1591(0.0052) 0.0056(0.0009) 0.0095(0.0010) 0.0009(0.0011) -0.0115(0.0006) -0.0114(0.0013) -0.0064(0.0013) -0.2407(0.0015) -0.1707(0.0042) -0.1704(0.0056) -0.0171(0.0009) -0.0117(0.0013) -0.0142(0.0015) -0.0220(0.0007) -0.308(0.0015) -0.0102(0.0017) -0.1715(0.0029) -0.2219(0.0136) -0.1995(0.0192) -0.0067(0.0031) -0.1226(0.0080) -0.0366(0.0121) -0.0736(0.0023) -0.1272(0.0085) -0.0630(0.0116)

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Confounding factors

Problem: important variables related to growth and FC left out \rightarrow biased estimates

Solution: augment regression with age, usually correlated with size; GOM=max{gross operating margins, 1}, availability of internal resources; ASSETS=net tangible assets, collaterals; to obtain Model 2A

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC} s_{t-1} + \beta_{1_{FC}} \ln(age_t) + \beta_{2_{FC}} \ln(GOM_{t-1}) + \beta_{3_{FC}} \ln(ASSETS_{t-1}) + \exp(\gamma_{FC} s_{t-1})\epsilon_{t_{FC}}$$

We include sectoral dummies following Pavitt taxonomy, Hall OReOP(2002) to obtain Model 2B

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$$s_t - s_{t-1} = c_{FC} + \lambda_{FC} s_{t-1} + \beta_{1_{FC}} \ln(age_t) + \beta_{2_{FC}} \ln(GOM_{t-1}) + \beta_{3_{FC}} \ln(ASSETS_{t-1}) + \exp(\gamma_{FC} s_{t-1})\epsilon_{t_{FC}}$$

We include sectoral dummies following Pavitt taxonomy, Hall OReOP(2002) to obtain Model 2B

	FC CLASS	Model 1	Model 2A	Model 2B
	NFC			
γ		-0.222*(0.001)	-0.207*(0.001)	-0.208*(0.001)
c		0.009*(0.001)	0.017*(0.001)	0.015*(0.001)
λ		-0.0007*(0.0003)	-0.008*(0.001)	-0.008*(0.001)
$ln(Age_{i,t})$			-0.023*(0.001)	-0.023*(0.001)
$ln(ASSETS_{i,t-1})$			0.021*(0.001)	0.020*(0.001)
$ln(GOM_{i,t-1})$			0.002(0.001)	0.002(0.001)
	MFC			
γ		-0.220*(0.001)	-0.205*(0.001)	-0.205*(0.001)
c		-0.011*(0.001)	0.003*(0.001)	-0.007*(0.001)
λ		-0.0076*(0.0003)	-0.018*(0.001)	-0.018*(0.001)
$ln(Age_{i,t})$			-0.040*(0.001)	-0.040*(0.001)
$ln(Assets_{i,t-1})$			0.028*(0.001)	0.027*(0.001)
$ln(GOM_{i,t-1})$			0.009*(0.001)	0.010*(0.001)
	UEC			
	<u>HFC</u>	0.1(1*(0.000)	0.142*(0.002)	0.1428(0.002)
γ		-0.161 (0.002)	-0.143 (0.002)	-0.143 (0.002)
c		$-0.013^{*}(0.003)$	$0.023^{*}(0.003)$	$0.014^{*}(0.003)$
λ		-0.030*(0.001)	-0.052*(0.002)	-0.052*(0.002)
$ln(Age_{i,t})$			-0.125*(0.003)	-0.127*(0.003)
$ln(Assets_{i,t-1})$			0.066*(0.003)	0.064*(0.003)
$\ln(\text{GOM}_{i,t-1})$			$0.019^{*}(0.002)$	$0.021^{*}(0.002)$

* significantly different from zero at 1%. Sandwich errors.

Sum up

In summary, we have shown that FC problems do have relevant effects on the operating activities of firms. In order to identify these effects, however, one has to do more work than just relying upon standard linear regression framework. FC effects are indeed manifold and impact on several aspects of firm growth dynamics, ranging well beyond a shift in the expected growth rates.

Data details

Data cleaned by Total Sales: for each firm a "nan" has been inserted instead of the original Total Sales value when the latter lied outside the interval

 $[Median(\log(TS_t))/10, Median(\log(TS_t)) * 10]$ $t = 1998, \dots, 2003$ Back.

Tried different assignment procedures:

- Lag 1: Based on the rating of the previous year
- The worst: Based on the worst rating obtained by the firm over the whole time window
- Persistent: Assigned only if a firm does not change its financial status over the whole time window

Results are qualitatively robust against assignment procedures. Back



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Robustness checks



MEDIAN FIRM SIZE AND AEP RIGHT WIDTH PARAMETER a, BY AGE CLASS

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Asymmetric Power Exponential distribution

$$f_{AEP}(x;\mathbf{p}) = \frac{1}{C} e^{-\left(\frac{1}{b_l} \left|\frac{x-m}{a_l}\right|^{b_l} \theta(m-x) + \frac{1}{b_r} \left|\frac{x-m}{a_r}\right|^{b_r} \theta(x-m)\right)}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m), \theta(x)$ is the Heaviside theta function and *C* the normalization constant. Back

 $b_l=2, b_r=\{5,1,0.5\}, a_l=1, a_r=1, m=0$

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Asymmetric Power Exponential distribution

$$f_{AEP}(x;\mathbf{p}) = \frac{1}{C} e^{-\left(\frac{1}{b_l} \left|\frac{x-m}{a_l}\right|^{b_l} \theta(m-x) + \frac{1}{b_r} \left|\frac{x-m}{a_r}\right|^{b_r} \theta(x-m)\right)}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m), \theta(x)$ is the Heaviside theta function and *C* the normalization constant. Back

 $b_l=2, b_r=\{5,1,0.5\}, a_l=1, a_r=1, m=0$ 0.4 0.35 h_ 0.3 0.25 0.2 0.15 0.1 0.05 0 -3 -2 -1 0 1 2 3 4 5 6

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Bo-Se-Ta Financial Constraints and Firm Dynamics

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