The evolution of the business cycles and growth-rates distributions

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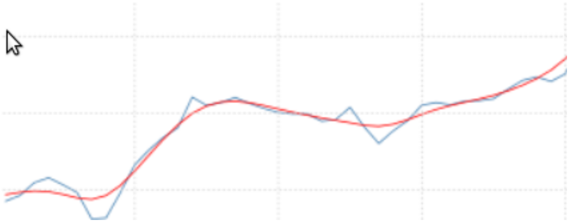
Outline

Motivation



- *High frequency fluctuations* have an important effect on the long-term performance of countries, (see for example Alesina et al., 1992; Ramey and Ramey, 1995; Fiaschi and Lavezzi, 2003; Koren and Tenreyro, 2007, among many others).

Motivation

- At the root of the empirical validation of any growth model lays the controversial issue of detrending. Cycles are commonly computed as deviations from an underlying trend

$$y_{i,t} = \theta_{i,t} + c_{i,t},$$

where $\theta_{i,t}$ is the trend, and $c_{i,t}$ the cycle (or residual).

- *Therefore, any decomposition of the time series in trend and cycles implies a partition of the economic information contained in the dynamics, assigning different weights to trend and cycle in term of explained variance.*
- The *business cycles stylized facts* are not robust, (Canova, 1998). In consequence, there is a lack of academic consensus of what constitutes cycles and discrepancies between statistically-based and economic-based approaches to detrend.

Motivation

- Growth-rates standard deviation is commonly used as a proxy to measure volatility of the economic fluctuations, providing useful information about the short term.
- Interestingly, in the cross section the PDF of GDP growth-rates exhibits heterskedasticity and large kurtosis (fat-tails), (see for example Canning et al., 1998; Lee et al., 1998; Castaldi and Dosi, 2009).
- But, fat-tails appears also for the residuals of detrended time series, for instance Fagiolo et al. (2008) and Fagiolo et al. (2009)

Methodology

We use a balanced panel of 91 countries for the 1960-2009 period using as unit of analysis the log of the gross domestic product ($y_{i,t} = \ln(\text{GDP})_{i,t}$) reported in the PWT 7.0.

Units of Analysis:

- First differences, or the growth-rates

$$r_{i,t} = y_{i,t} - y_{i,t-1}.$$

- Hodrick-Prescott filter

$$\min_{[\theta_t]_{t=1}^{T}} \left\{ \sum_{t=1}^{T} c_{i,t}^2 + \lambda \sum_{t=2}^{T} ((\theta_{i,t+1} - \theta_{i,t}) - (\theta_{i,t} - \theta_{i,t-1}))^2 \right\},$$

HP-Filter, Examples

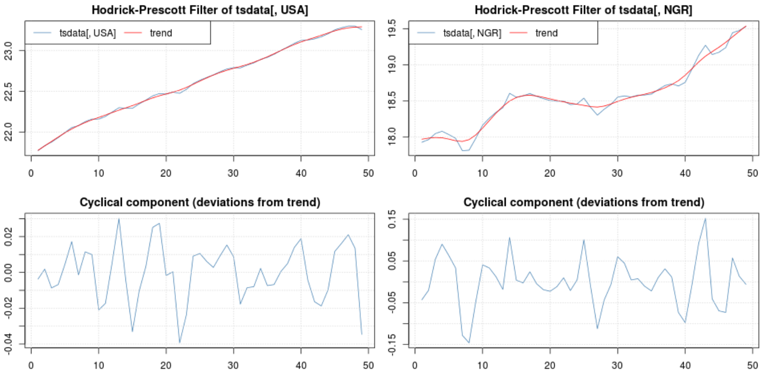


Figure: HP-filtering on USA and Nigeria's GDP time series

Autocorrelations (ACF)

Growth-rates and HP-cycles time series are commonly autocorrelated, for instance:

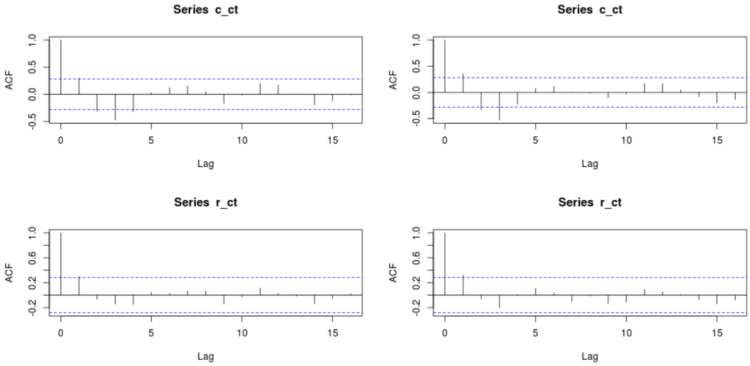


Figure: Correlograms for USA and Nigeria, HP-cycles (up), and growth-rates (down)

Some statistics

Vanishing std-dev, but frequent excess of kurtosis jumps!

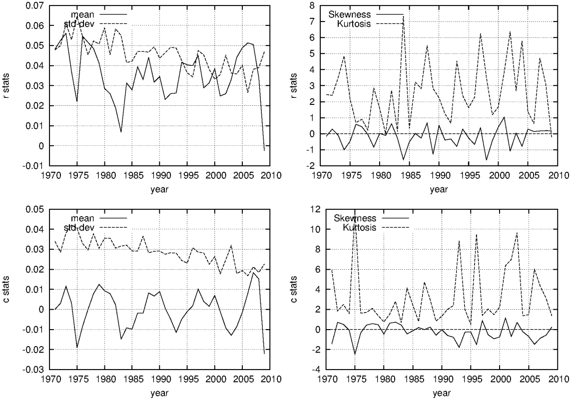


Figure: First four moments, growth-rates (up), and HP-cycles (down)

Remember how looks like the GDP distribution

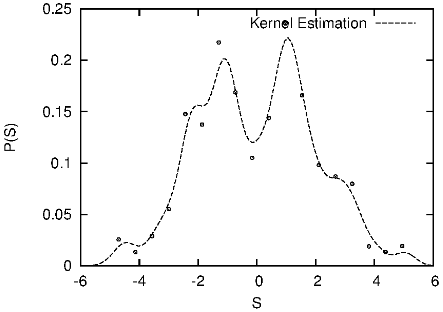


Figure: Full Sample

where size is defined as

$$S_{i,t} = y_{i,t} - \overline{y_t}.$$

PDFs of economic fluctuations

Both PDF are heteroscedastic

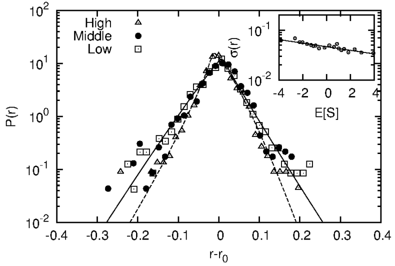


Figure: Empirical PDF of growth-rates (left) and HP-cycles (rigth) for different country income classes; in the subplot volatility vs. the average of country sizes using bin statistics.

Volatility, Power Law relation

$$\ln(\sigma_x) \sim \beta_x S.$$

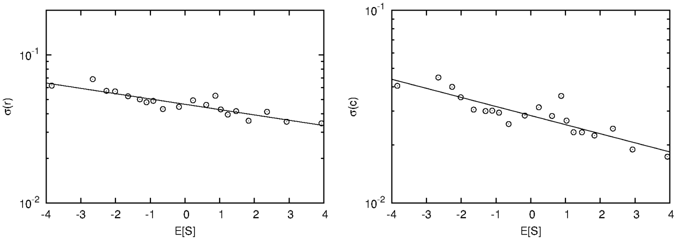


Figure: Bin statistics between std-dev of economic fluctuation vs. country size. For growth-rates $\beta_r = -0.082(0.012)$, while for HP-cycles $\beta_c = -0.108(0.012)$.

How can we remove this effect to get homoscedastic distributions?

Econometric Model

Let's assume that

$$x_{i,t} = \alpha_\tau + \phi_\tau x_{i,t-1} + u_{i,t}, \tag{1}$$

where α_τ is a constant term, ϕ_τ the autoregressive term, the expected value of u is zero, and the subscript τ makes reference to the pooled interval of years. Then, if $u_{i,t} = e^{\beta_\tau S_{i,t}} \varepsilon_{i,t}$, we get

$$\varepsilon_{i,t} = \frac{x_{i,t} - \phi_\tau x_{i,t-1} - \alpha_\tau}{e^{\beta_\tau S_{i,t}}}. \tag{2}$$

Notice that the distribution of ε is equivalent to the distribution of the rescaled economic fluctuation, which is our main objective.

Econometric Model

One can solve the equation (2) using the Minimum Absolute Deviation (MAD) method,

$$\{\beta_\tau, \phi_\tau, \alpha_\tau\} = \arg\min_{\beta,\phi,\alpha} \sum_i \sum_{t \in \tau} \left| \frac{x_{i,t} - \phi x_{i,t-1} - \alpha}{e^{\beta S_{i,t}}} \right|, \qquad (3)$$

where the expression is proportional to the log-likelihood function when ε is Laplace distributed.

Subbotin Distribution

Once the MAD equation is solved, we fit the rescaled fluctuation PDF. We do this using a class of asymmetric exponential power (AEP) family of densities,

$$f(x, a, b, m) = \begin{cases} \frac{1}{A} e^{-\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}} & x < m \\ \frac{1}{A} e^{-\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}} & x > m \end{cases} \tag{4}$$

where

$$A = a_l b_l^{1/b_l} \Gamma(1 + 1/b_l) + a_r b_r^{1/b_r} \Gamma(1 + 1/b_r),$$

where, $a_{\{l,r\}}$ characterize the variance, $b_{\{l,r\}}$ the shape of the tails, and m is the position of the mode, (Bottazzi, 2004).
The symmetric version of the density is recovered if the left and right parameters are equal. For instance, the Normal and Laplace distributions are obtained with $a_l = a_r$ and $b_l = b_r = 2$ and $b_l = b_r = 1$, respectively.

Results, Growth-rates

Growth-rates, pooled sample

Parameters	Non-scaled	Bin-scaled	MAD-scaled
β	-	-0.082	-0.113
	-	(0.012)	(0.006)
ϕ	-	-	0.339
	-	-	(0.011)
α	-	-	0.026
	-	-	(0.001)
Subbotin estimation			
b	1.074	1.129	1.087
	(0.030)	(0.032)	(0.031)
b_l	0.984	1.034	1.011
	(0.039)	(0.044)	(0.042)
b_r	1.194	1.257	1.183
	(0.052)	(0.057)	(0.052)

* Standard errors are reported in parenthesis

Table: Estimated parameters of the stochastic process and Subbotin paramters of rescaled growth rates, for the period $(1960, 2009)$, using OLS on the binned statistics and the MAD regression

Results, HP-cycles

HP-cycles, pooled sample

Parameters	Non-scaled	Bin-scaled	MAD-scaled
β	-	-0.108	-0.109
	-	(0.012)	(0.006)
ϕ	-	-	0.304
	-	-	(0.011)
α	-	-	0.001
	-	-	(0.000)
Subbotin estimation			
b	1.041	1.124	1.071
	(0.029)	(0.032)	(0.031)
b_l	0.877	0.948	1.007
	(0.033)	(0.039)	(0.041)
b_r	1.2	1.294	1.158
	(0.048)	(0.055)	(0.051)

* Standard errors are reported in parenthesis

Table: Estimated parameters of the stochastic process and Subbotin paramters of rescaled HP-cycles, for the period $(1960, 2009)$, using OLS on the binned statistics and the MAD regression.

Rescaled Distributions, before and after

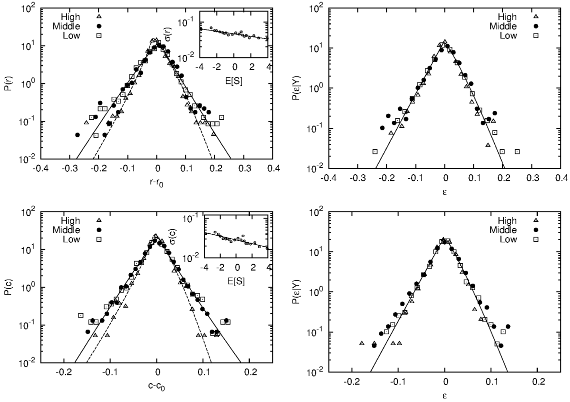


Figure: Rescaled PDF, notice how country classes converge in shape

Dynamic Approach

- Notice that our MAD estimator approach is more accurate and therefore does not rely on the use of many years.
- A potential problem when pooling many years (e.g. fifty years) is that one might mix different macro phenomena from different periods, as for instance technology shocks, spread of crisis, changes in policies, etc.
- we use moving windows of ten years, hence, in every realization there are 91×10 observations available.

Dynamic Results Growth-rates

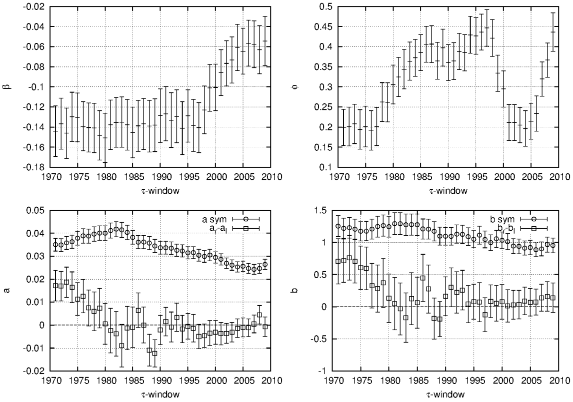


Figure: Scaling β parameter, autoregressive ϕ parameter, Subbotin shape parameters, symmetric b and asymmetric comparison $b_r - b_l$, and estimated Subbotin variance parameters, symmetric a and asymmetric comparison $a_r - a_l$. Error bars correspond to two times $+/-$ the standard deviation of estimations.

Dynamic Results HP-cycles

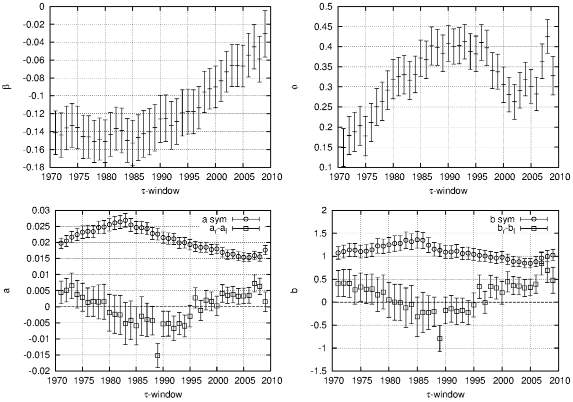


Figure: Scaling β parameter, autoregressive ϕ parameter, Subbotin shape parameters, symmetric b and asymmetric comparison $b_r - b_l$, and estimated Subbotin variance parameters, symmetric a and asymmetric comparison $a_r - a_l$. Error bars correspond to two times $+/-$ the standard deviation of estimations.

Conclusions

- we contribute to the empirical analysis of the distribution of the international GDP business cycles and its evolution.
- we found out heteroskedastic fat-tailed probability density functions under different specifications of cycles
- Observed heterogeneity in the residuals is observed under different filter specifications. Suggesting an ubiquitous presence of correlating mechanisms that survive aggregation from firms to sectors to countries (Castaldi and Dosi, 2009).
- The dynamic analysis showed that distribution tails evolved getting fatter, suggesting an increasing non null probability of finding high amplitude fluctuations.