NOTES for STATISTICS

- Estimator for σ_Y^2

$$\tilde{S}_Y^2 = \frac{1}{n} \sum_{1}^{n} (Y_i - \bar{Y})^2 = \frac{1}{n} \sum_{1}^{n} (Y_i^2 - \bar{Y}^2)$$

$$E[\tilde{S}_Y^2] = E\left[\frac{1}{n} \sum_{1}^{n} (Y_i^2 - \bar{Y}^2)\right] = E\left[\frac{1}{n} \sum_{1}^{n} Y_i^2\right] - E\left[\frac{1}{n} \sum_{1}^{n} \bar{Y}^2\right] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - E\left[\frac{1}{n}\, n\, \bar{Y}^2\right] = \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - E[\bar{Y}^2] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - E\left[\left(\frac{1}{n} \sum_{1}^{n} Y_i\right)^2\right] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} E\left[\left(\sum_{1}^{n} Y_i\right)^2\right] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} E\left[\sum_{1}^{n} Y_i^2 + \sum_{1}^{n} \sum_{\substack{1 \\ i \neq j}}^{n} Y_i Y_j\right] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} E\left[\sum_{1}^{n} Y_i \sum_{\substack{1 \\ i \neq j}}^{n} Y_j\right] =$$

$$= \frac{1}{n} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} \left[\sum_{1}^{n} \sum_{\substack{1 \\ i \neq j}}^{n} \mu_Y^2\right] =$$

$$= \frac{n-1}{n^2} \sum_{1}^{n} E[Y_i^2] - \frac{1}{n^2} (n^2 - n)\, \mu_Y^2 =$$

$$= \frac{n-1}{n^2}\, n \left(\sigma_Y^2 + \mu_Y^2\right) - \frac{n(n-1)}{n^2} \mu_Y^2 =$$

$$= \left(\frac{n-1}{n}\right) \sigma_Y^2 + \left(\frac{n-1}{n}\right) \mu_Y^2 - \left(\frac{n-1}{n}\right) \mu_Y^2 = \frac{n-1}{n} \sigma_Y^2$$

$$S_Y^2 = \frac{n}{n-1} \tilde{S}_Y^2 = \frac{1}{n-1} \sum_{1}^{n} (Y_i - \bar{Y})^2$$

$$E[S_Y^2] = E\left[\frac{n}{n-1} \tilde{S}_Y^2\right] = \frac{n}{n-1} \frac{n-1}{n} \sigma_Y^2 = \sigma_Y^2$$

- Estimator for σ_Y^2 when μ_Y is known

$$S_Y^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \cancel{2}\mu_Y)^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i^2 - \mu_Y^2)$$

$$E[S_Y^2] = E\left[\frac{1}{n}\sum_{i=1}^{n} Y_i^2 - \mu_Y^2\right] = \frac{1}{n}\sum_{i=1}^{n} E[Y_i^2] - \mu_Y^2 =$$

$$= \frac{1}{n}\, n\, (\sigma_Y^2 + \mu_Y^2) - \mu_Y^2 = \sigma_Y^2$$