NOTES for PROBABILITY

- $E[Y] = \sum_1^k{}_i \, Y_i \Pr(Y = Y_i) = \sum_1^k{}_i \, y_i \, P_i =$

$$= \sum_1^m{}_i (a + b x_i) P_i = \sum_1^m{}_i (a p_i + b x_i p_i) =$$

$$= \sum_1^m{}_i a p_i + \sum_1^m{}_i b x_i p_i = a \sum_1^m{}_i p_i + b \sum_1^m{}_i x_i p_i =$$

$$= a + b\, E[X]$$

$E[\cdot]$ is a linear operator

- $VAR[Y] = \sum_1^m{}_i (y_i - E[Y])^2 p_i = \sum_1^m{}_i ((a + b x_i) - a - b E[x])^2 p_i$

$$= \sum_1^m{}_i (a + b x_i - a - b E[x])^2 p_i =$$

$$= \sum_1^m{}_i (b x_i - b E[x])^2 p_i = \sum_1^m{}_i b^2 (x_i - E[x])^2 p_i =$$

$$= b^2 \sum_1^m{}_i (x_i - E[x])^2 p_i = b^2 \; VAR[x]$$

- SEE BELOW for $E[x^2]$ **

$VAR[\cdot]$ is a quadratic operator

- For discrete r.v. the Law of Iterated Expectations (from now on LIE) is

$$E[E[Y|X]] = E[Y]$$

$$E[E[Y|X]] = E\left[\sum_1^k{}_j \, y_j P(Y = y_j | X)\right] =$$

$$= E\left[\sum_1^k{}_j \, y_j \, P(Y = y_j | X)\right] =$$

$$= \sum_1^h{}_i \left(\sum_1^k{}_j \, y_j \, P(Y = y_j | X = x_i)\right) P(X = x_i) =$$

$$= \sum_1^h{}_i \sum_1^k{}_j \, y_j \, P(Y = y_j | X = x_i) \, P(X = x_i) =$$

$$= \sum_{1}^{h}{}_{i} \sum_{1}^{k}{}_{j} \; y_j \; P(Y=y_j \mid X=x_i) \; P(X=x_i) =$$

$$= \sum_{1}^{k}{}_{j} \; y_j \sum_{1}^{h}{}_{i} \; P(Y=y_j \mid X=x_i) \; P(X=x_i) =$$

$$= \sum_{1}^{k}{}_{j} \; y_j \sum_{1}^{h}{}_{i} \; P(Y=y_j \, , X=x_i) =$$

$$= \sum_{1}^{k}{}_{j} \; y_j \; P(Y=y_j) = \sum_{1}^{k}{}_{j} \; y_j P_j = E[Y]$$

$$E[X^2] = \sum_{1}^{m}{}_{i} \; x_i^2 \, P_i = VAR[X] + (E[X])^2$$

$$VAR[X] = \sum_{1}^{m}{}_{i} \; (x_i - E[X])^2 P_i = \sum_{1}^{m}{}_{i} \; (x_i^2 + E[X]^2 - 2x_i E[x_i]) P_i =$$

$$= \sum_{1}^{m}{}_{i} \; x_i^2 P_i + E[X]^2 \sum_{1}^{m}{}_{i} \; P_i - 2E[x_i] \sum_{1}^{m}{}_{i} \; x_i P_i =$$

$$= \sum_{1}^{m}{}_{i} \; x_i^2 P_i + E[X]^2 - 2\,E[X]\,E[X] =$$

$$= \sum_{1}^{m}{}_{i} \; x_i^2 P_i - E[X]^2 \quad \Rightarrow \quad VAR[X] = E[X^2] - E[X]^2$$

$$\sigma_x^2 = E[X^2] - \mu_x^2$$

$$E[X^2] = \sigma_x^2 + \mu_x^2$$