IMETRICS Cheat-Sheet

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Statistics and Probability

Basics

$$\mathrm{Pr}(x,y) = \mathrm{Pr}(X = x, Y = y) \quad \text{[joint]}$$
$$\mathrm{Pr}(y) = \sum_i \mathrm{Pr}(X = x_i, Y = y) \quad \text{[marginal]}$$
$$\mathrm{Pr}(y|x) = \frac{\mathrm{Pr}(X = x, Y = y)}{\sum_j \mathrm{Pr}(X = x, Y = y_j)} \text{[conditional]}$$

Law of iterated expectations

$$E(Y) = \sum_i E(Y|X = x_i)\mathrm{Pr}(X = x_i) = E[E(Y|X)]$$

Mean square error (MSE)

Let θ be an unknown parameter and $T_n(\theta)$ an estimator for it.

$$\mathrm{MSE}(T_n(\theta)) = E[(T_n(\theta) - \theta)^2]$$

Sums of random variables

$$E(a + bX + cY) = a + b\mu_X + c\mu_Y$$
$$\mathrm{var}(a + bX) = b^2\sigma_X^2$$
$$\mathrm{var}(aX + bY) = a^2\sigma_X^2 + 2ab\sigma_{XY} + b^2\sigma_Y^2$$
$$\mathrm{var}(Y_1 + \ldots + Y_n) = \sum_i \mathrm{var}(Y_i) + \sum_i \sum_{j\neq i} \mathrm{cov}(Y_i, Y_j)$$
$$E(Y^2) = \sigma_Y^2 + \mu_Y^2 \quad E(XY) = \sigma_{XY} + \mu_X\mu_Y$$
$$\mathrm{cov}(a + bX + cR, Y) = b\sigma_{XY} + c\sigma_{RY}$$

Mean and Variance estimators

Y is a r.v. with mean and variance μ_Y and σ_Y^2

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \quad E(\bar{Y}) = \mu_Y \quad \mathrm{var}(\bar{Y}) = \frac{\sigma_Y^2}{n}$$
$$\bar{\sigma}_Y^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 \quad E(\bar{\sigma}_Y^2) = \frac{n-1}{n}\sigma_Y^2$$
$$S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 \quad E(S_Y^2) = \sigma_Y^2$$

P-value

Y is a r.v. with mean and variance μ_Y and σ_Y^2

$$H_0 : E(Y) = \mu_0 \qquad H_1 : E(Y) \neq \mu_0$$
$$p - \mathrm{value} = \mathrm{Pr}_{H_0}\left[|\bar{Y} - \mu_0| > |\bar{Y}^{act} - \mu_0|\right]$$

Econometrics

OLS estimators

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X}$$
$$\hat{\beta}_1 = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_i (X_i - \bar{X})^2} = \frac{\mathrm{COV}(X,Y)}{\mathrm{VAR}(X)}$$

Variance of OLS estimators

$$\sigma^2_{\hat{\beta}_0} = \frac{1}{n}\frac{\mathrm{var}(H_i u_i)}{[E(H_i^2)]^2} \quad \text{with } H_i = 1 - \frac{\mu_X}{E[X_i^2]}X_i \ ,$$
$$\sigma^2_{\hat{\beta}_1} = \frac{1}{n}\frac{\mathrm{var}[(X_i - \mu_x)u_i]}{[\mathrm{var}(X_i)]^2} \ .$$

Standard errors of OLS estimators

$$SE(\hat{\beta}_0) = \sqrt{\frac{1}{n}\frac{\frac{1}{n-2}\sum_i \hat{H}_i^2\hat{u}_i^2}{\frac{1}{n}\left[\sum_i \hat{H}_i^2\right]^2}} \quad \text{with } \hat{H}_i = 1 - \left(\frac{\bar{X}}{\frac{1}{n}\sum_i X_i^2}\right)X_i$$
$$SE(\hat{\beta}_1) = \sqrt{\frac{1}{n}\frac{\frac{1}{n-2}\sum_i (X_i - \bar{X})^2\hat{u}_i^2}{\left[\frac{1}{n}\sum_i (X_i - \bar{X})^2\right]^2}}$$