EXERCISES in STATISTICS

1.

$$\bar{Y} = \frac{1}{n}\sum_{1}^{n}{}_i Y_i$$

$Y_0 \quad \mu_Y \quad \sigma_Y^2$

$\quad\;\; E[Y] \quad VAR[Y]$

$$\hat{Y} = Y_1$$

$$\tilde{Y} = \frac{1}{n}\left(\frac{1}{2}Y_1 + \frac{3}{2}Y_2 + \dots + \frac{1}{2}Y_{n-1} + \frac{3}{2}Y_n\right) \qquad n \text{ is even}$$

We start by checking if they are unbiased

$$E[\bar{Y}] = E\left[\frac{1}{n}\sum_{1}^{n}{}_i Y_i\right] = \frac{1}{n}\sum_{1}^{n}{}_i E[Y_i] = \frac{1}{n} n \mu_Y = \mu_Y \quad \checkmark$$

$$E[\hat{Y}] = E[Y_1] = \mu_Y \quad \checkmark$$

$$E[\tilde{Y}] = E\left[\frac{1}{n}\left(\frac{1}{2}Y_1 + \frac{3}{2}Y_2 + \dots + \frac{1}{2}Y_{n-1} + \frac{3}{2}Y_n\right)\right] =$$

$$= \frac{1}{n}\left(\frac{1}{2}E[Y_1] + \frac{3}{2}E[Y_2] + \dots + \frac{1}{2}E[Y_{n-1}] + \frac{3}{2}E[Y_n]\right] =$$

$$= \frac{1}{n}\left(\frac{1}{2}\frac{n}{2}\mu_Y + \frac{3}{2}\frac{n}{2}\mu_Y\right) = \frac{1}{n}\frac{n}{2}\mu_Y\frac{4}{2} = \mu_Y \quad \checkmark$$

We compute their VAR

$$VAR[\bar{Y}] = VAR\left[\frac{1}{n}\sum_{1}^{n}{}_i Y_i\right] \overset{iid}{=} \frac{1}{n^2}\sum_{1}^{n}{}_i VAR[Y_i] = \frac{1}{n^2} n \sigma_Y^2 = \frac{\sigma_Y^2}{n}$$

$$VAR[\hat{Y}] = \sigma_Y^2$$

$$VAR[\tilde{Y}] = \frac{1}{n^2}\left[\frac{1}{4}VAR[Y_1] + \frac{9}{4}VAR[Y_2] + \dots + \frac{1}{4}VAR[Y_{n-1}] + \frac{9}{4}VAR[Y_n]\right] =$$

$$= \frac{1}{n^2}\left[\frac{1}{4}\frac{n}{2}\sigma_Y^2 + \frac{9}{4}\frac{n}{2}\sigma_Y^2\right] = \frac{1}{n^2}\frac{n}{2}\sigma_Y^2\frac{10}{4} = \frac{\sigma_Y^2}{n}\frac{5}{4}$$

Note that $VAR[\bar{Y}]$ and $VAR[\tilde{Y}]$ for $n \to \infty$ go to 0.

$\bar{Y}$ is consistent

$\tilde{Y}$ is consistent

$\hat{Y}$ is inconsistent